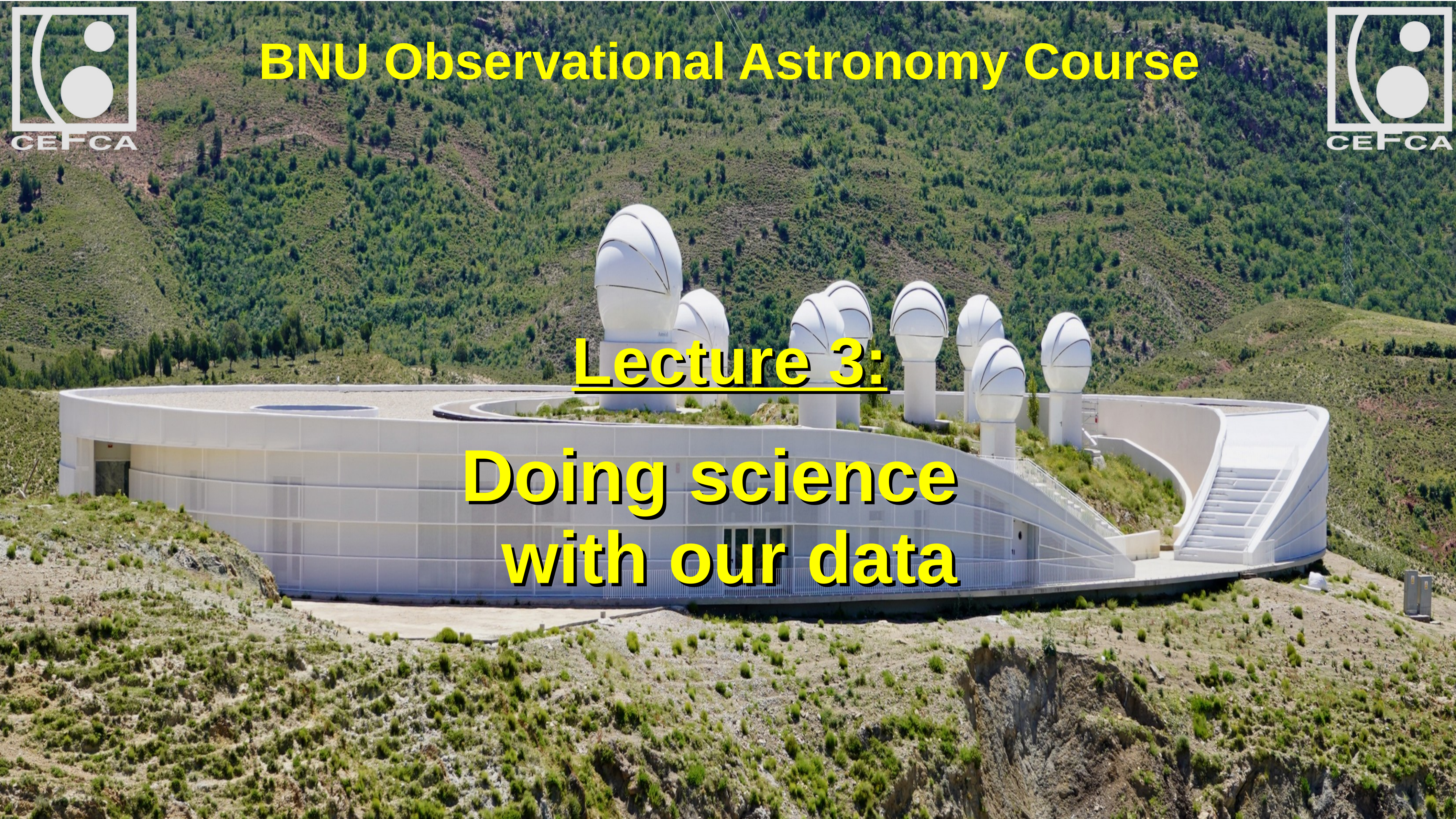


BNU Observational Astronomy Course

Lecture 3:

Doing science with our data

CONTENTS

- **PART 1:** Basic definitions: luminosity, brightness, apparent and absolute magnitude, magnitude scales

- **PART 2:** Introducing some Physics: Planck’s law, spectra, physical properties, astronomical filters

- **PART 3:** The big picture: doing science

Defining

the basics

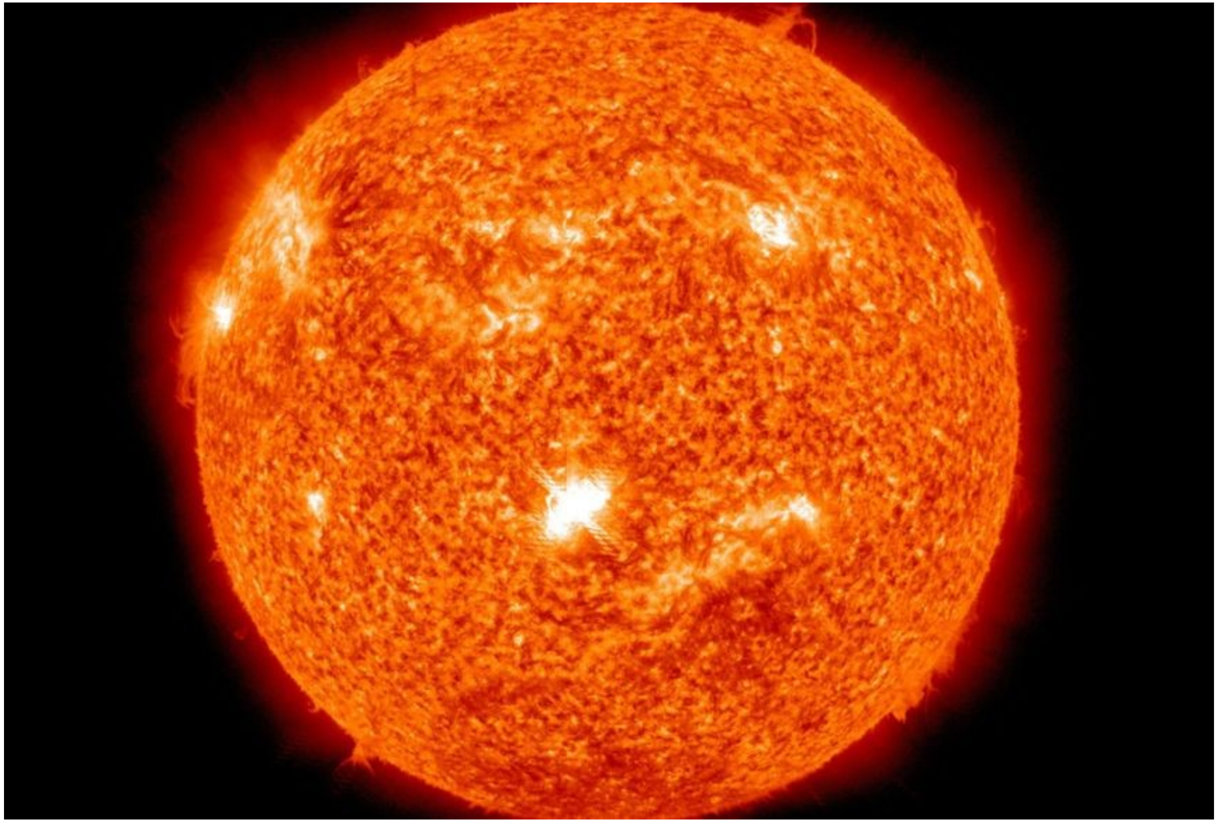
Astronomy

- Astronomy is a very unique science, in the sense that in the majority of cases we *cannot* interact directly with our objects of study, or perform a lab experiment in the traditional sense.

- The only thing we can do is *observe* the light coming to us from celestial objects and try to understand the information this light carries with it.

- The purpose of observational astronomy is to capture (= telescopes), record (= CCD) and analyse (= photometry & spectroscopy) this light, and then use theoretical models and/or simulations to explain celestial objects and phenomena.

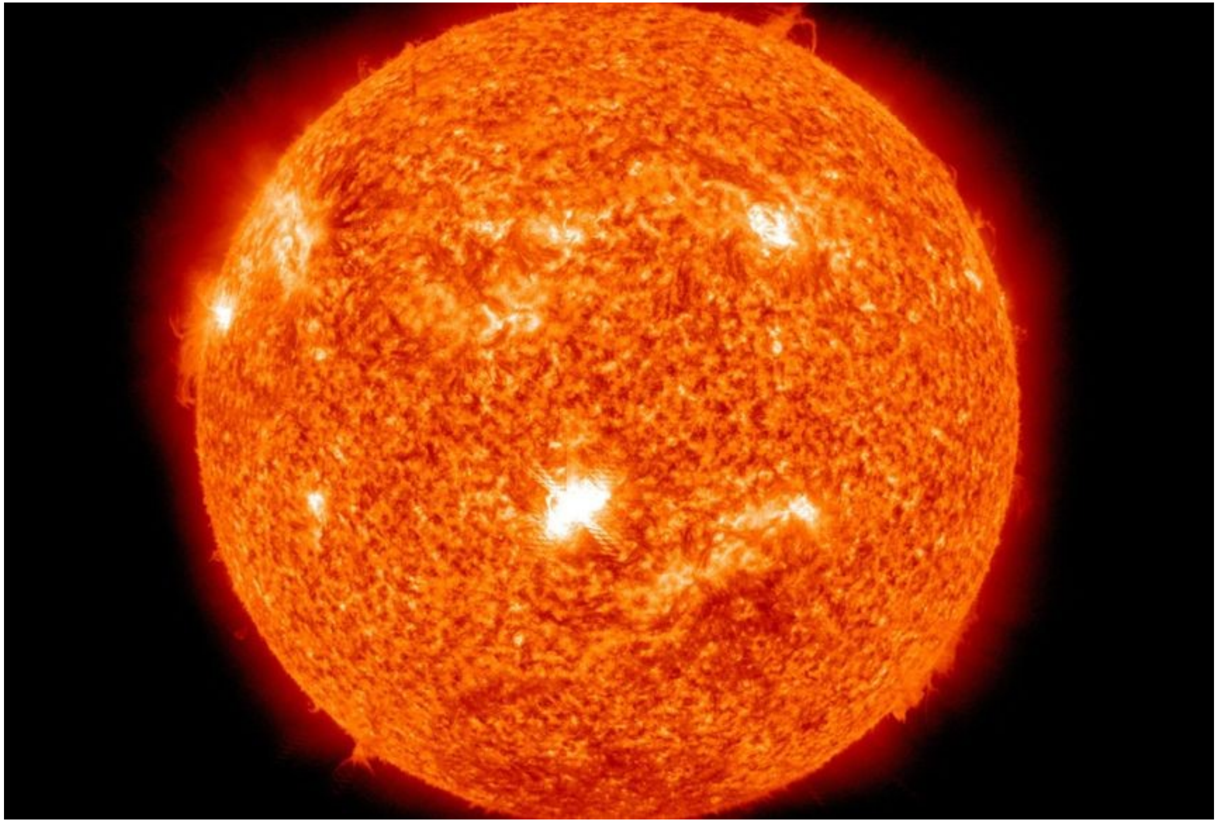
Luminosity



Luminosity is defined as the total amount of radiant energy emitted over all wavelengths per unit time in all directions.

You can think of luminosity as the *power* of a source. In fact, its unit is the Watt [W].

Luminosity



For a star, its luminosity is:

$$L = 4\pi R^2 \sigma T_{eff}^4$$

where

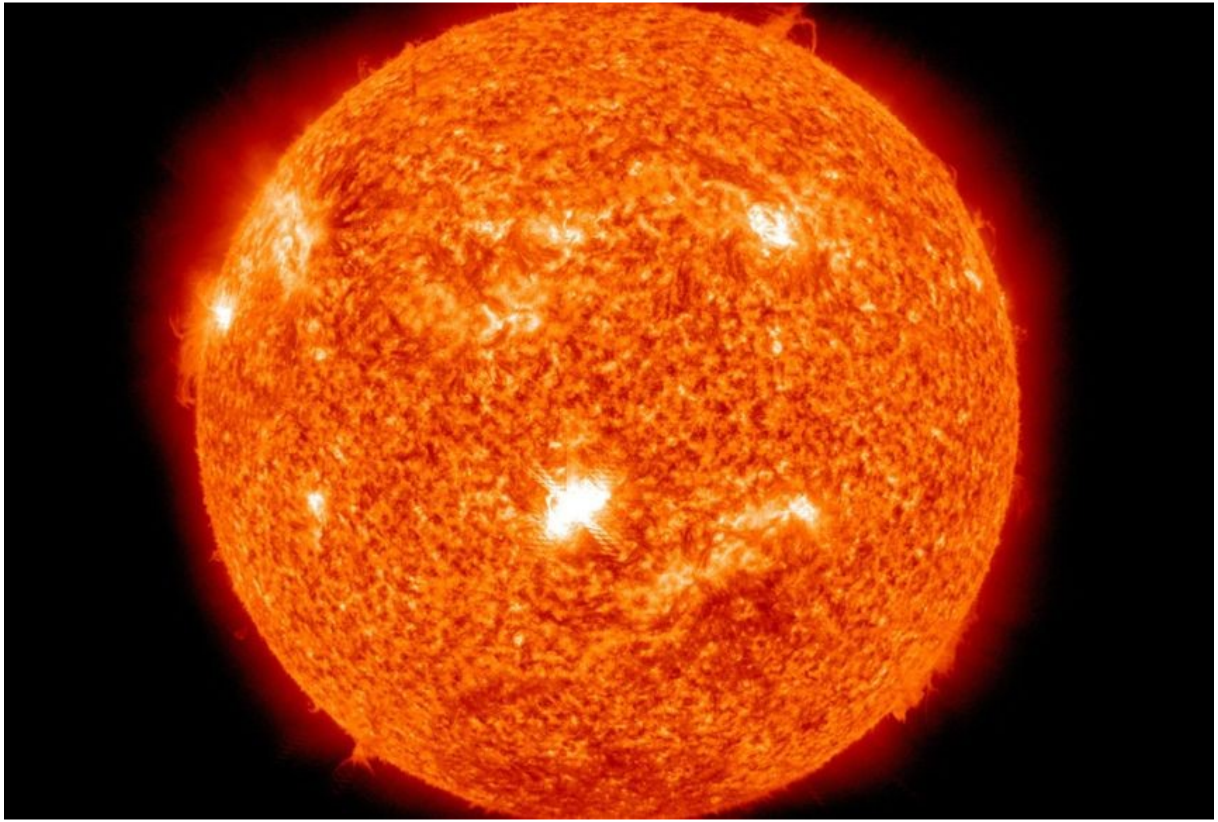
R – the radius [m]

$\mathbf{T_{eff}}$ – the effective temperature [K]

σ – the Stefan-Boltzmann constant ~5.67 x 10^{-8} [$Wm^{-2}K^{-4}$]

Luminosity is an *absolute, intrinsic* quantity and depends only on the physical parameters!

Brightness

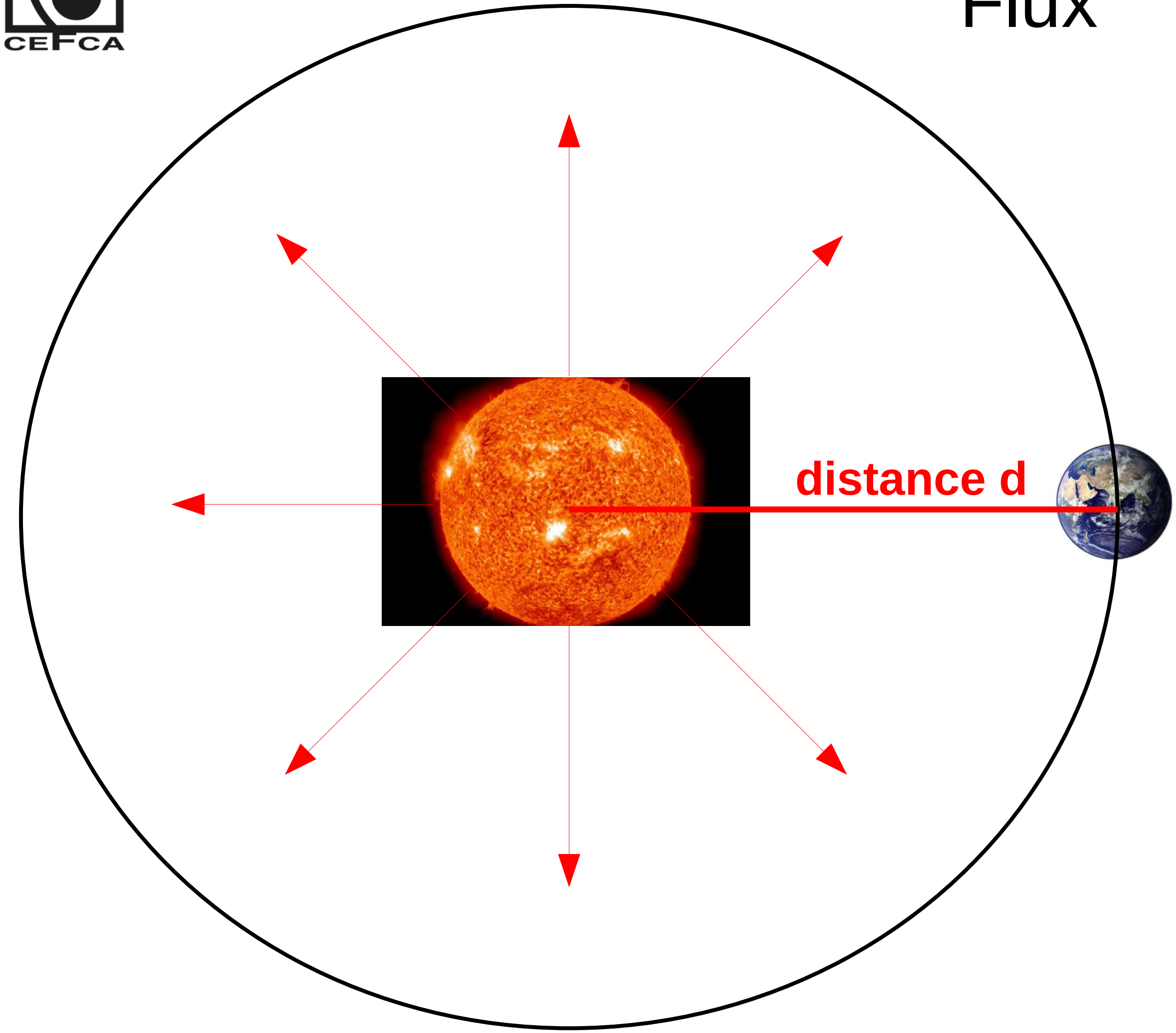


We use the term ***brightness*** to describe, how bright an object *appears* to be, as seen from Earth.

Brightness is an extrinsic, relative quantity, as it can be affected by many external factors.

The Sun is the *brightest* star in the sky, but it is certainly **not** the most *luminous.*

Flux



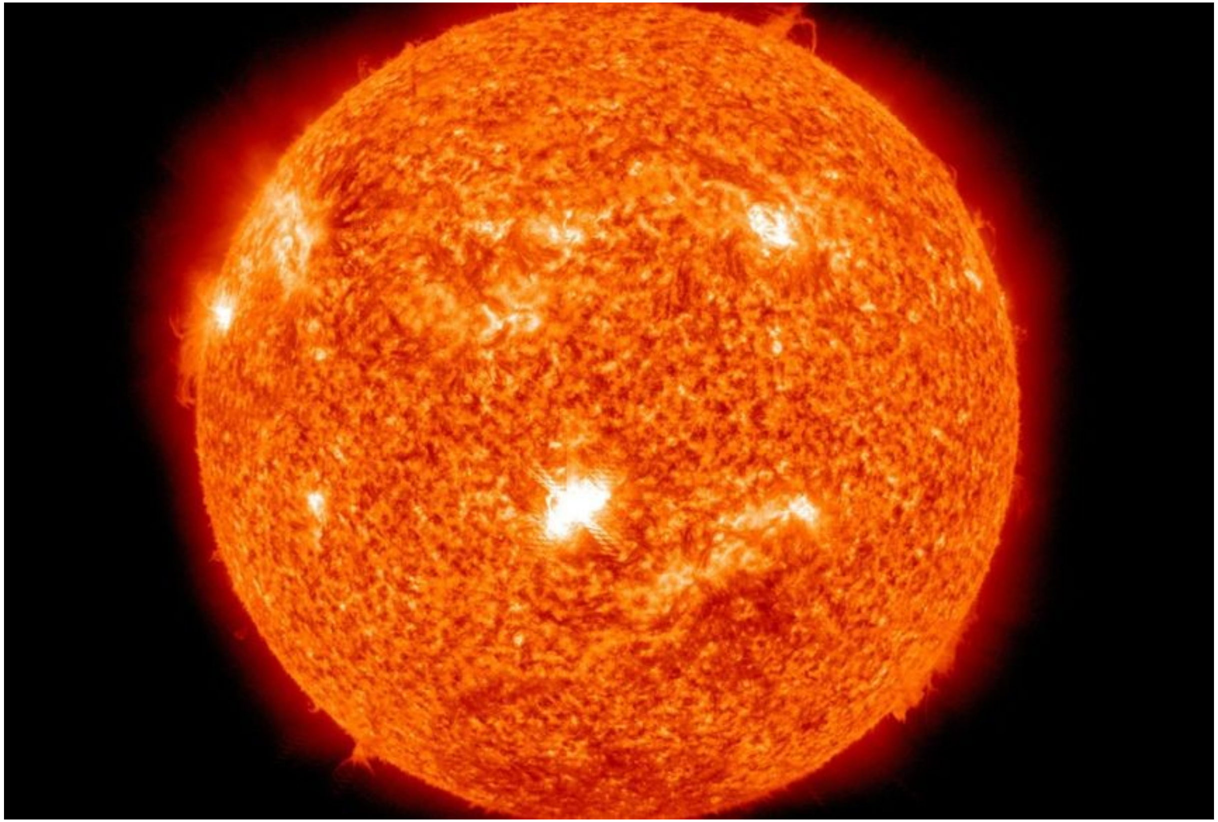
Only a very small portion of the energy emitted by a star is actually captured and recorded when we observe this star.

We call this portion ***flux***, and for a star at distance ***d*** from the Earth, its flux is given by:

$$F = \frac{L}{4\pi d^2}$$

Notice how flux can be connected to brightness...

Magnitude



Magnitudes are a convenient way to refer to an object’s *brightness*. In this case, we also refer to them as *apparent* magnitudes.

Magnitudes are just numbers, they do not have any unit of measurement attached to them.

Magnitude

OBJECT	MAGNITUDE
Sun	-26.74
Moon (full)	-12.6
Venus (max)	-4.92
Jupiter (max)	-2.94
Sirius (α CMa)	-1.46
Canopus (α Car)	-0.74
Rigil Kentaurus (α Cen)	-0.27
Arcturus (α Boo)	-0.05
Vega (α Lyr)	0.03
Capella (α Aur)	0.08
Rigel (β Ori)	0.13
Procyon (α CMi)	0.34

The Greek astronomer Hipparchos of Nicea (~ 190-120 BCE) is commonly credited with the creation of the magnitude scale.

Hipparchos divided the stars he could see in 6 groups, which he called magnitudes.

The stars of magnitude **1** were the **brightest**, the ones of magnitude **6** the **faintest**.

The magnitude scale is inverted!

Magnitude

OBJECT	MAGNITUDE
Sun	-26.74
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The English astronomer Norman Pogson formalised the magnitude scale in 1856.

Pogson defined that “a star of magnitude **6** should be precisely **100** times **fainter** than a star of magnitude **1**”.

In Pogson’s scale, a change of one magnitude corresponds to a change in brightness of $100^{1/5}$ = **2.512**

The magnitude equation

We can formalise the definition of Pogson with the following Equation

$$\boxed{\frac{F_1}{F_2} = 100^{-\frac{m_1 - m_2}{5}}}$$

For $m_1 = 1$ and $m_2 = 6$, we have:

$$\frac{F_1}{F_2} = 100^{-\frac{1-6}{5}} = 100^{-\frac{-5}{5}} = 100$$

This allows us to obtain the **magnitude equation**

$$\boxed{m_1 - m_2 = -2.5 * \log_{10}\left(\frac{F_1}{F_2}\right)}$$

Proof

$$\frac{F_1}{F_2} = 100^{-\frac{m_1 - m_2}{5}} \Leftrightarrow \log_{10}\left(\frac{F_1}{F_2}\right) = \log_{10} 100^{\left(-\frac{m_1 - m_2}{5}\right)} \Leftrightarrow$$

1 $\boxed{\log_{10}(A^X) = X * \log_{10}(A)}$

1

$$\Leftrightarrow \log_{10}\left(\frac{F_1}{F_2}\right) = -\frac{1}{5}(m_1 - m_2) \log_{10}(100) \Leftrightarrow \log_{10}\left(\frac{F_1}{F_2}\right) = -\frac{1}{5}(m_1 - m_2) \log_{10}(10^2)$$

1

$$\Leftrightarrow \log_{10}\left(\frac{F_1}{F_2}\right) = -\frac{2}{5}(m_1 - m_2) \underbrace{\log_{10}(10)}_{1} \Leftrightarrow \boxed{m_1 - m_2 = -\frac{5}{2} * \log_{10}\left(\frac{F_1}{F_2}\right)}$$

Magnitude scales

The magnitude equation $m_1 - m_2 = -2.5\log_{10}\left(\frac{F_1}{F_2}\right)$ connects a ***ratio of fluxes***, which we can ***measure*** with our observations, with a ***difference in magnitudes***. However, it does not give us the individual values of m_1 or m_2, only their difference.

The following pairs of (m_1,m_2) values, all satisfy $m_1 - m_2 = 5$: (25,20); (20,15); (15,10); (10,5)…

In order to obtain the actual m_1 and m_2 values, we need a reference point, so we have to create what is called a ***magnitude scale***.

Vega-magnitude scale

For this magnitude scale the star ***Vega*** is taken as reference and, by definition has $\mathbf{m_{Vega} = 0}$.

$$m_{Star} - m_{Vega} = -2.5 \log_{10}\left(\frac{F_{Star}}{F_{Vega}}\right) \Leftrightarrow m_{Star} = -2.5 \log_{10}\left(\frac{F_{Star}}{F_{Vega}}\right)$$

Vega is the 5th brightest star in the sky. By definition of the Vega-scale, the four stars that are *brighter* than Vega (Sirius, Canopus, Rigil Kentaurus and Arcturus) will have negative m_{Star}.

AB-magnitude scale

AB means “absolute”, because in this scale, magnitudes are calculated on an absolute level, and not a relative one, i.e. with respect to another star, like the Vega-system.

$$m_{AB} = -2.5 \log_{10} f_{\nu} - 48.6$$

where f_ν is the spectral flux density in a frequency interval (ν_1,ν_2): $f_{\nu} = \int_{\nu_1}^{\nu_2} F_{\nu}(\nu) \, d\nu$

The scale is designed so that $\mathrm{mag}_{AB} = \mathrm{mag}_V$, the magnitude in the V-band (more about this soon) and the constant number, 48.6, ensures that $\mathrm{mag}_{AB,Vega} = \mathrm{mag}_{V,Vega} = 0$.

Apparent magnitude, brightness and luminosity

Apparent magnitudes and fluxes are relative quantities connected to the ***brightness***.
If we have two stars, Star1 and Star2, and if Star1 is ***brighter*** than Star2, then

$$m_1 < m_2 \Leftrightarrow m_1 - m_2 < 0 \Leftrightarrow -2.5\log_{10}\left(\frac{F_1}{F_2}\right) < 0 \Leftrightarrow \log_{10}\left(\frac{F_1}{F_2}\right) > 0 \Leftrightarrow \frac{F_1}{F_2} > 1 \Leftrightarrow F_1 > F_2$$

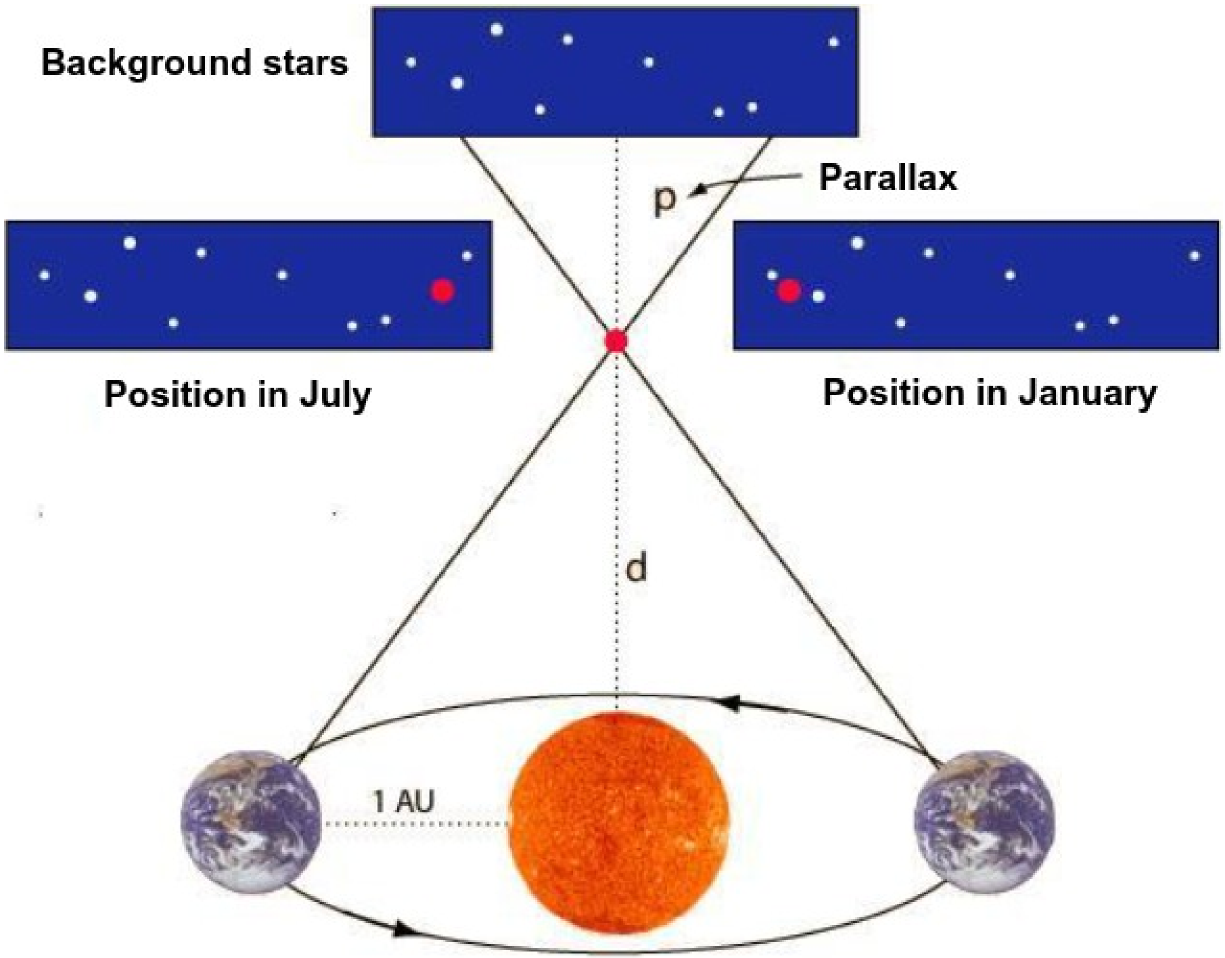
However, we ***cannot*** directly connect them to **luminosity**, because we do ***not*** know the ***distance***.

$$F_1 > F_2 \Leftrightarrow \frac{L_1}{4\pi d_1^2} > \frac{L_2}{4\pi d_2^2} \cancel{\Leftrightarrow L_1 > L_2}$$

Example: $L_1 = 1*L_{sun}$ and $d_1 = 1$ pc
$L_2 = 10*L_{sun}$ and $d_2 = 100$ pc

$$\left.\begin{matrix} \\ \end{matrix}\right\} \frac{L_1}{4\pi d_1^2} > \frac{L_2}{4\pi d_2^2} \quad \textbf{BUT} \quad L_1 < L_2$$

Distances



Distances of objects in our Galaxy can be measured via the ***parallax*** method.

An object with parallax of 1 arcsec is defined to be at a distance of 1 ***parsec*** (***par***allax ***sec***ond). One pc Is equal to ~3.26 light years.

$$\boxed{d[pc] = \frac{1}{\varpi\,[arcsec]} = \frac{1000}{\varpi\,[mas]}}$$

mas = milli-arcsecond = 10^{-3} arcsec

Currently, the *Gaia* space mission is providing parallaxes, and therefore distances, for billions of objects.

Absolute Magnitude

We can now define the ***absolute magnitude M*** as
“the apparent magnitude a star *would* have if located at a distance of 10 parsec”.

$$M_1 < M_2 \Leftrightarrow \ldots \Leftrightarrow F_{M,1} > F_{M,2} \Leftrightarrow \frac{L_1}{4\pi 10^2} > \frac{L_2}{4\pi 10^2} \Leftrightarrow L_1 > L_2$$

The equation relating the apparent magnitude **m** with the absolute magnitude **M**:

$$\boxed{m - M = 5 * \log_{10}(d\,[pc]) - 5}$$

In astronomy, the quantity **m-M** is called the ***distance modulus***.

Proof

$$m_1 - m_2 = -2.5 * \log_{10}\left(\frac{F_1}{F_2}\right) \Leftrightarrow m_1 - M_1 = -2.5 * \log_{10}\left(\frac{F_1}{F_{M,1}}\right) \Leftrightarrow$$

$$\Leftrightarrow m_1 - M_1 = -2.5 * \log_{10}\left(\frac{\frac{L_1}{4\pi d_1^2}}{\frac{L_1}{4\pi 10^2}}\right) \Leftrightarrow m_1 - M_1 = -2.5 * \log_{10}\left(\frac{\cancel{L_1}\ \cancel{4\pi}\ 10^2}{\cancel{L_1}\ \cancel{4\pi}\ d_1^2}\right) \Leftrightarrow$$

$$\Leftrightarrow m_1 - M_1 = -2.5 * \log_{10}\left(\frac{10}{d_1}\right)^2 \Leftrightarrow m_1 - M_1 = -2.5 * \log_{10}\left(\frac{d_1}{10}\right)^{-2} \overset{1}{\Leftrightarrow}$$

1 $\boxed{\log_{10}(A^X) = X * \log_{10}(A)}$

2 $\boxed{\log_{10}\left(\frac{A}{B}\right) = \log_{10}(A) - \log_{10}(B)}$

$$\Leftrightarrow m_1 - M_1 = 5 * \log_{10}\left(\frac{d_1}{10}\right) \overset{2}{\Leftrightarrow} m_1 - M_1 = 5 * \left(\log_{10}(d_1) - \cancelto{1}{\log_{10}(10)}\right) \Leftrightarrow \boxed{m - M = 5\log_{10}(d) - 5}$$

We need more information...

We saw that by comparing the absolute magnitude M of two stars,
we can say which of these two stars is more ***luminous***.

$$M_1 < M_2 \Leftrightarrow \ldots \Leftrightarrow F_{M,1} > F_{M,2} \Leftrightarrow \frac{L_1}{4\pi 10^2} > \frac{L_2}{4\pi 10^2} \Leftrightarrow L_1 > L_2$$

However, we are missing information in order to be able
to compare the physical properties of these stars

$$L_1 > L_2 \Leftrightarrow 4\pi R_1^2 \sigma T_{1,eff}^4 > 4\pi R_2^2 \sigma T_{2,eff}^4$$

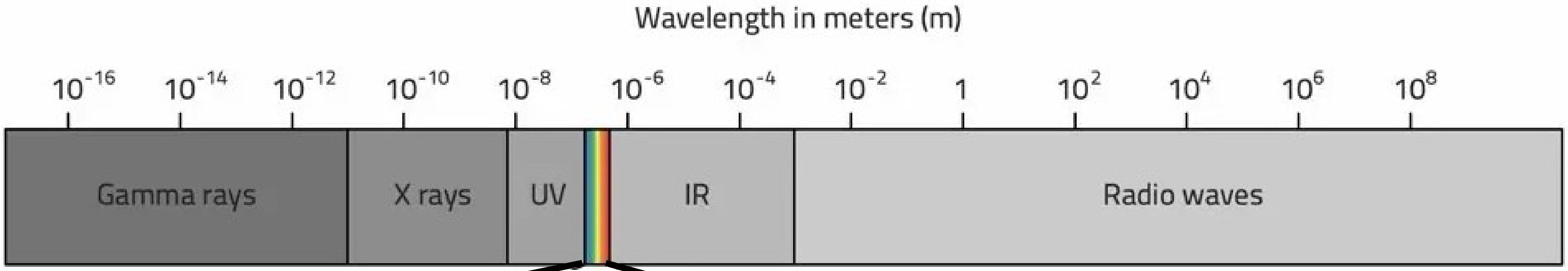
R_1 **???** R_2

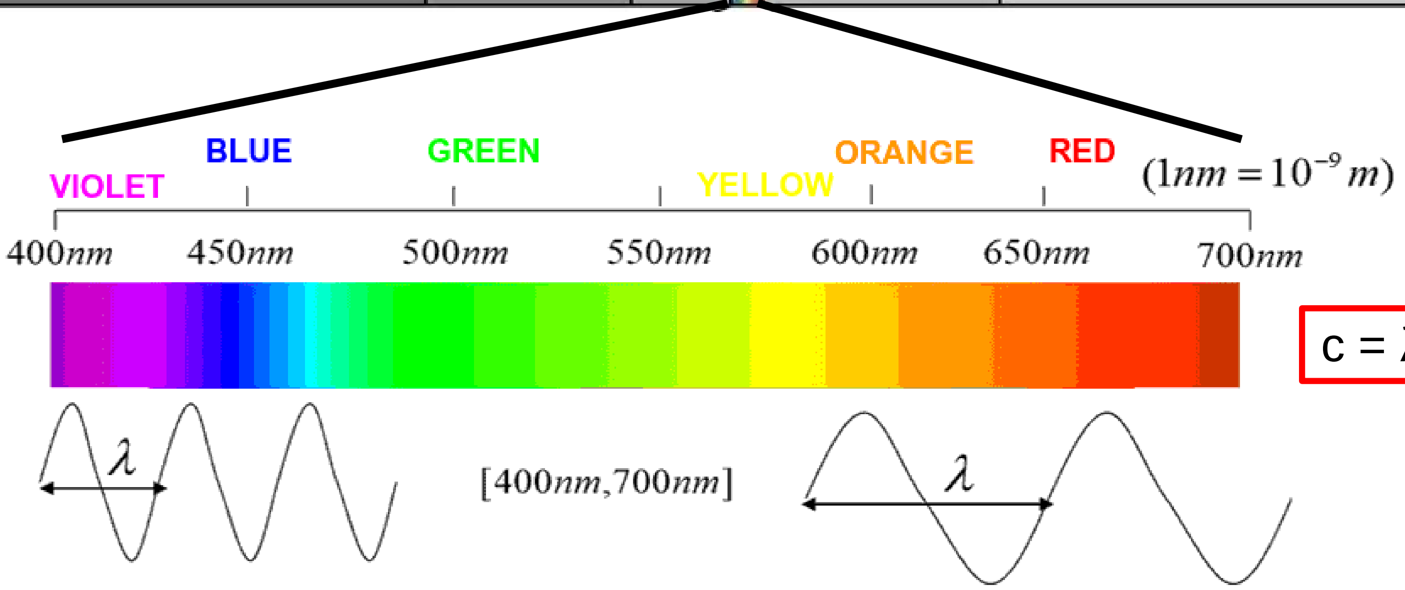
$T_{1,eff}$ **???** $T_{2,eff}$

Introducing

some Physics

The electromagnetic spectrum

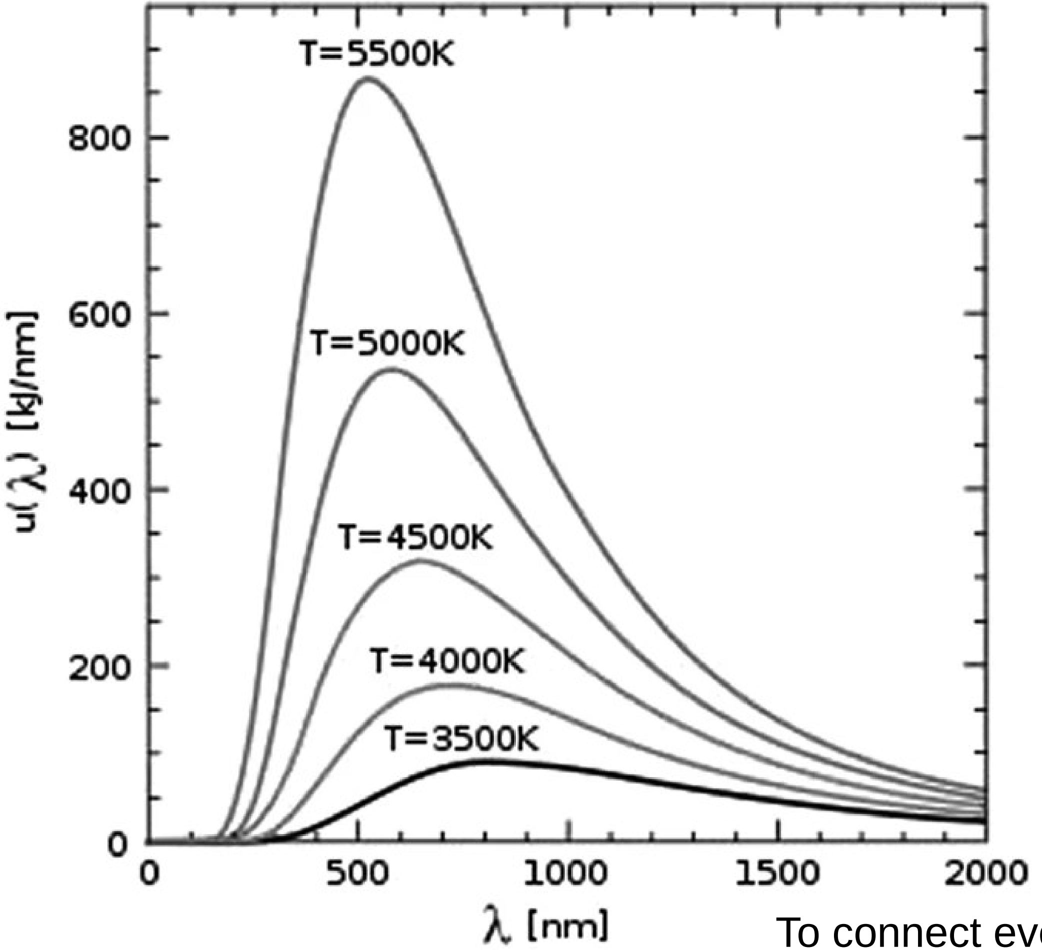




$$\boxed{c = \lambda \cdot \nu}$$

Color	Wavelength (nm)
violet	380–450
blue	450–485
cyan	485–500
green	500–565
yellow	565–590
orange	590–625
red	625–750

Planck's Law



Stars emit radiation as ***black bodies,*** following ***Planck's Law***.

$$B_\nu(\nu,T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{\left(\frac{h\nu}{k_B T}\right)}-1}$$

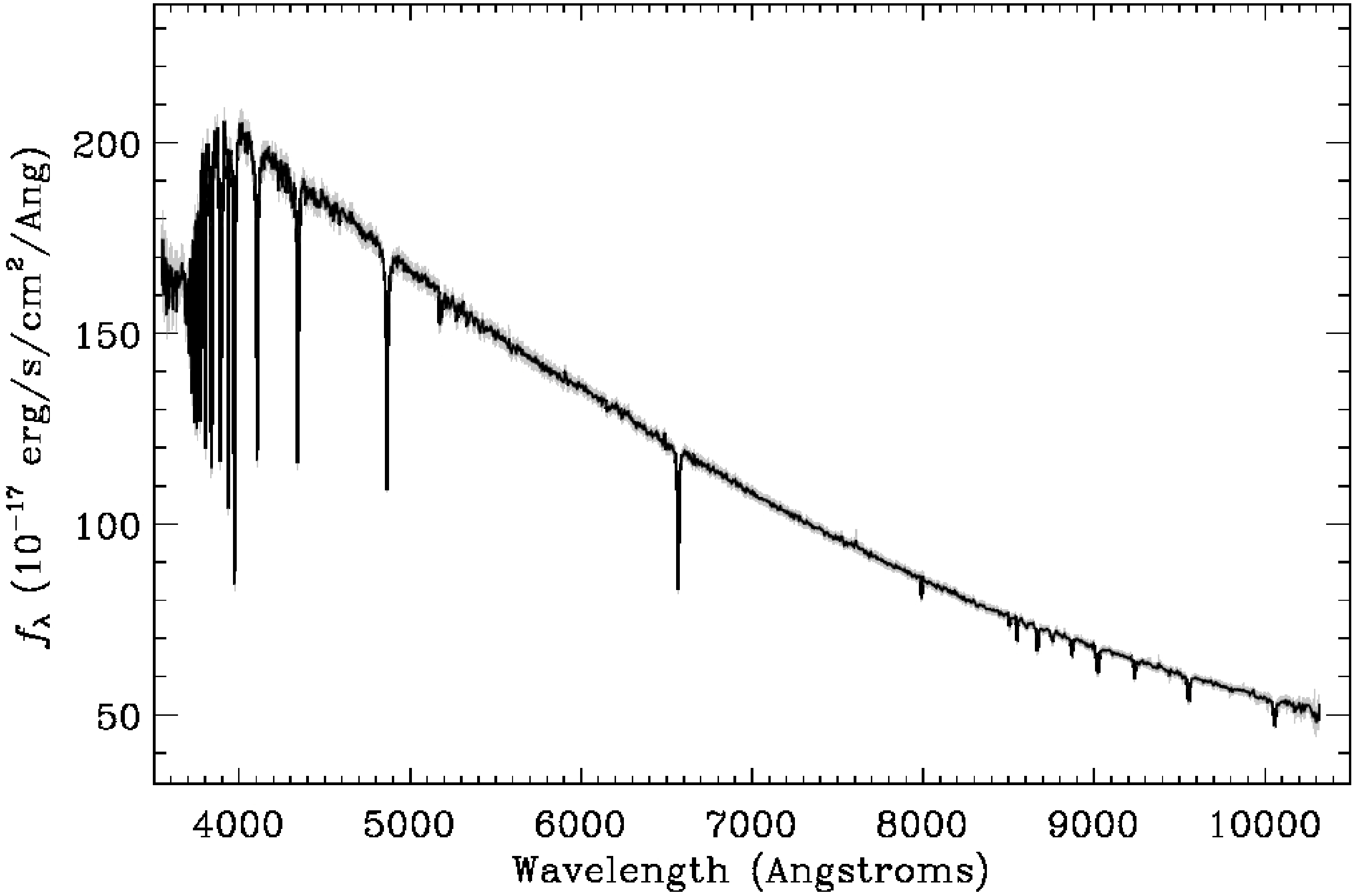
where:

ν – the frequency [Hz]
T – the temperature [K]
h – the Planck constant [$6.63\text{x}10^{-34}$ JHz^{-1}]
k_B – the Boltzmann constant [$1.38\text{x}10^{-23}$ JK^{-1}]
c – the speed of light [ms^{-1}]

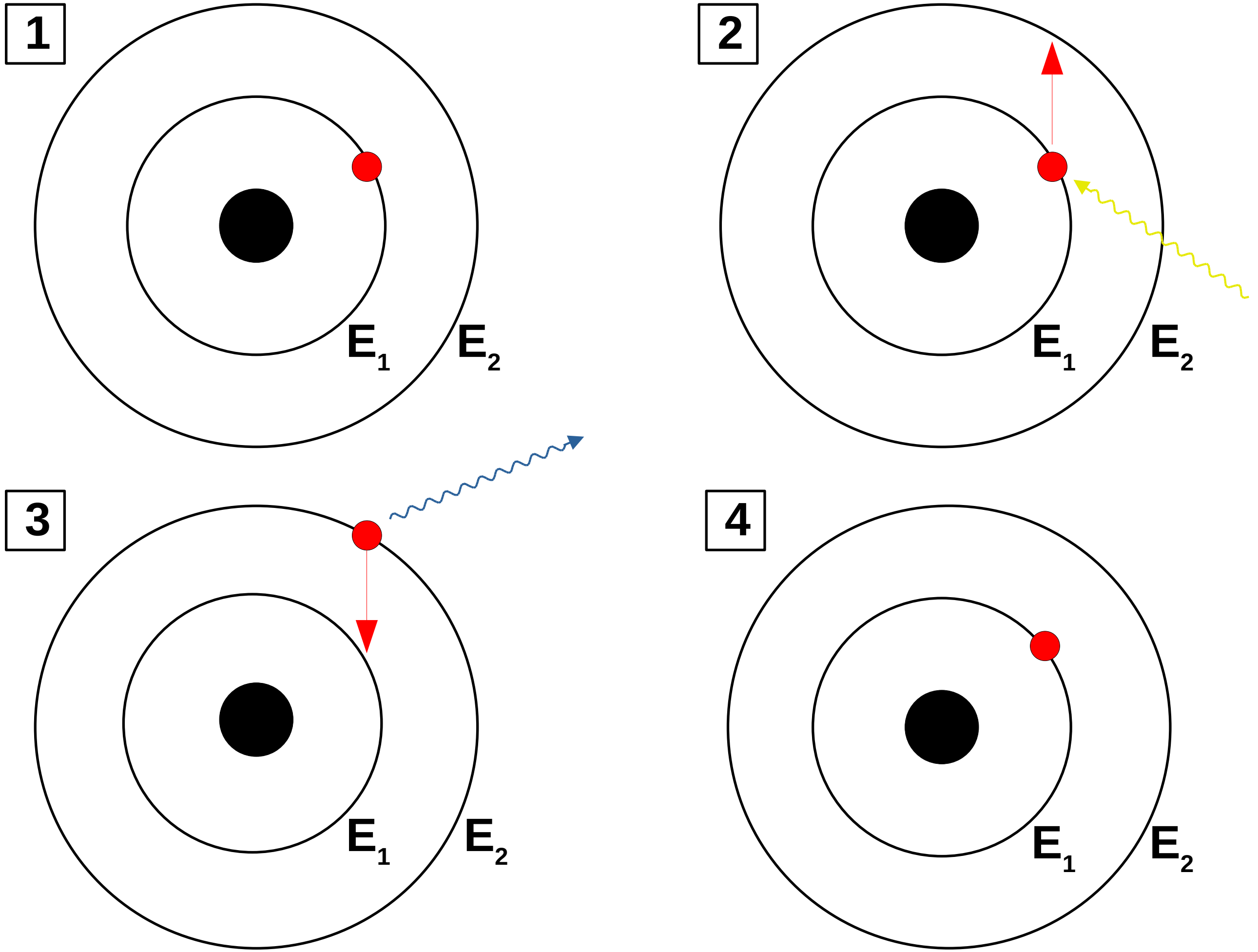
Notice how the equation depends on both *frequency* and ***temperature***.

To connect everything: $\int_0^\infty d\nu \int_0^{\pi/2} d\theta \int_0^{2\pi} d\varphi\, B_\nu(T)\cos(\theta)\sin(\theta) = \sigma T^4$

The spectrum of a star



Spectral lines

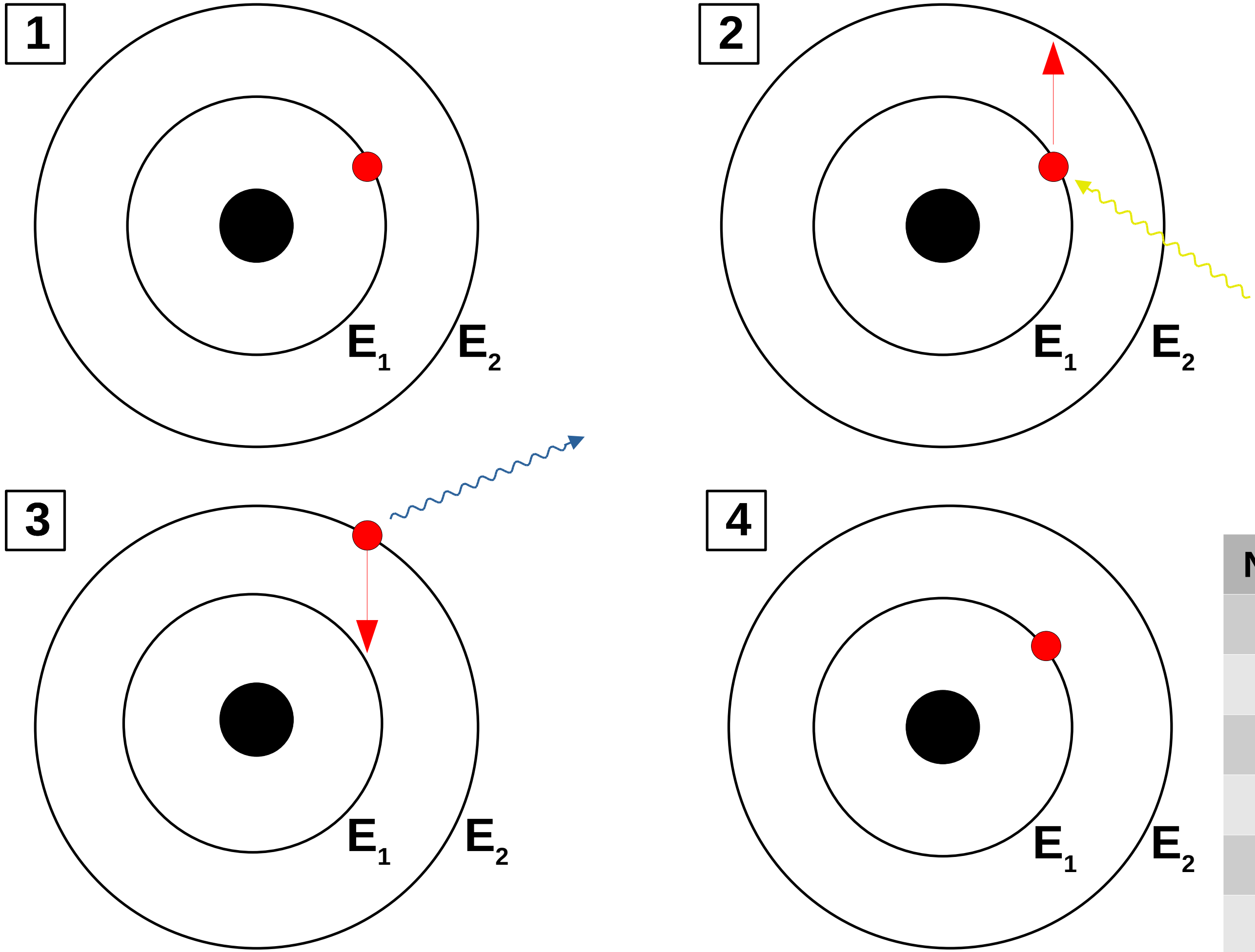


1) An **electron** is in the ground state with energy E_1.

(2-3) An incoming **photon** interacts with the electron. If $\mathbf{E_{ph} = E_2 - E_1}$, the electron will be *excited* and jump to the first excited state E_2.

(3-4) Very quickly, the electron will be *de-excited* and will return to the ground state, **emitting a photon** to get rid of the excess energy.

Spectral lines



The energy levels are specific and well defined for each element.

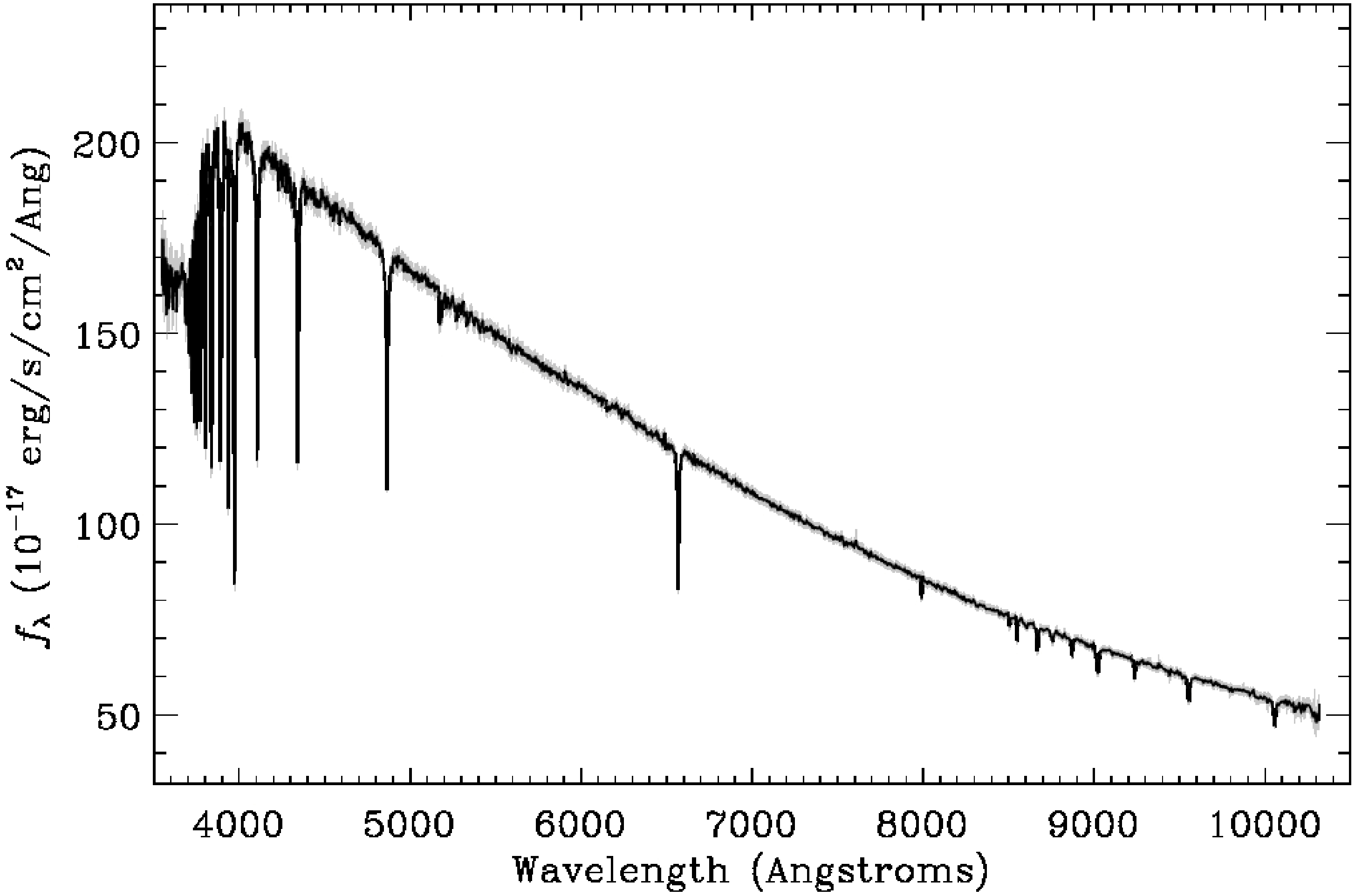
Photons emitted by this process will therefore have very specific and well defined energy:

$$\Delta E = E_{n+1} - E_n = h \cdot \nu = \frac{hc}{\lambda}$$

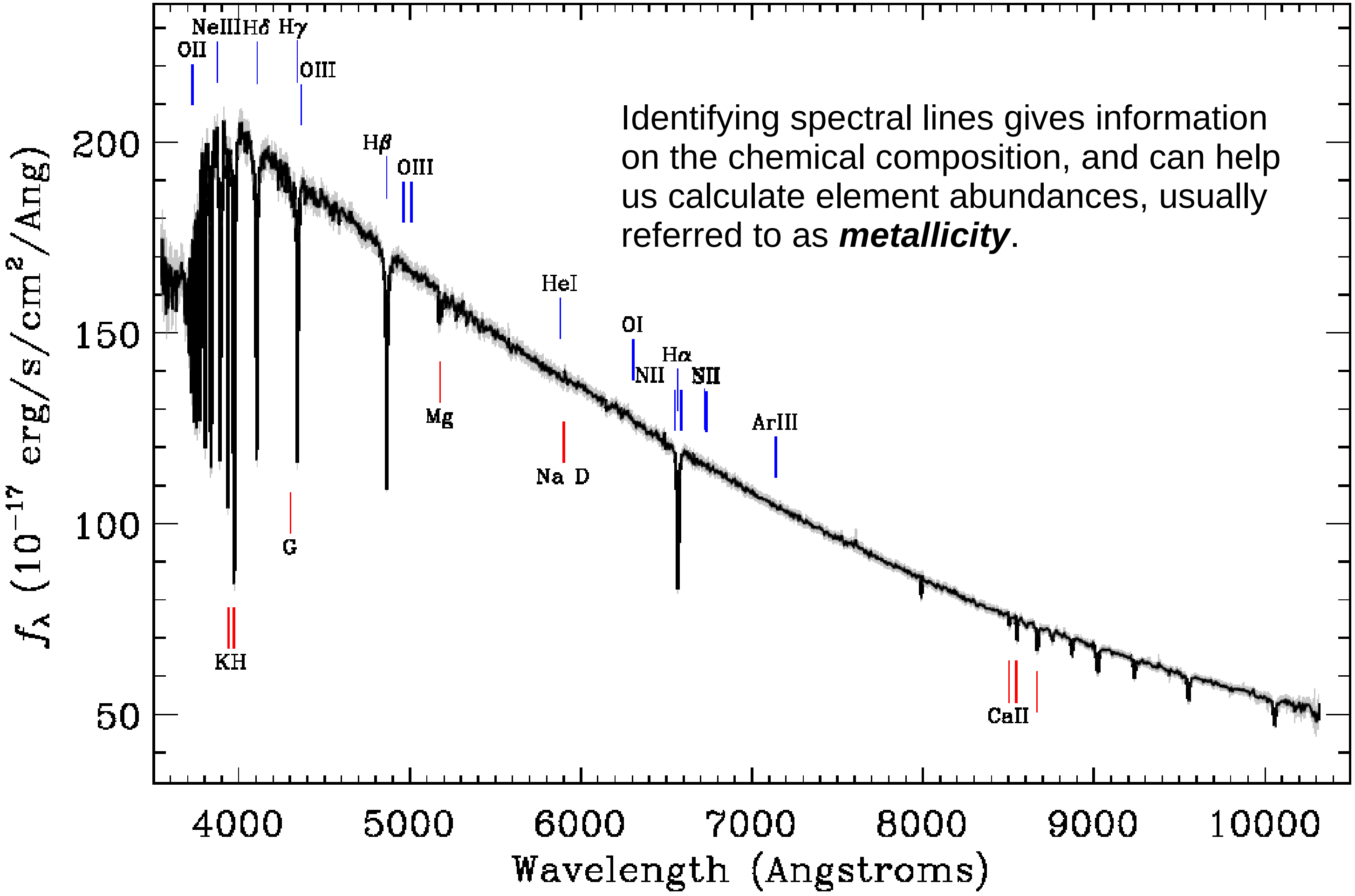
Hydrogen Balmer series

NAME	TRANSITION	WAVELENGTH [Ang]
Hα	3 → 2	6562
Hβ	4 → 2	4861
Hγ	5 → 2	4340
Hδ	6 → 2	4101
Hε	7 → 2	3970
H8	8 → 2	3889

The spectrum of a star

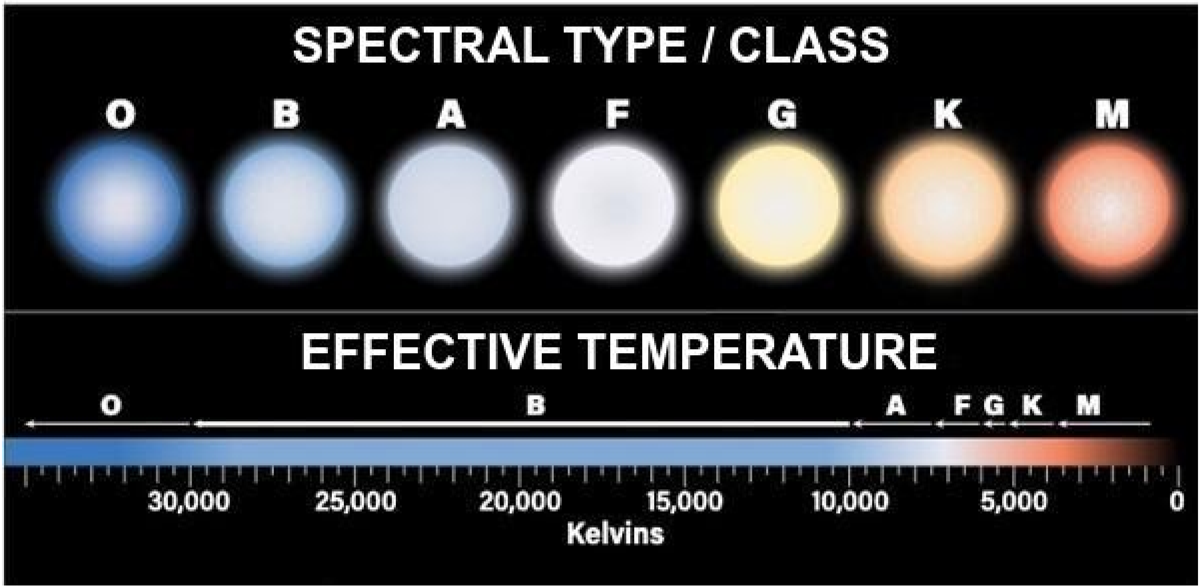


The spectrum of a star



Identifying spectral lines gives information on the chemical composition, and can help us calculate element abundances, usually referred to as ***metallicity***.

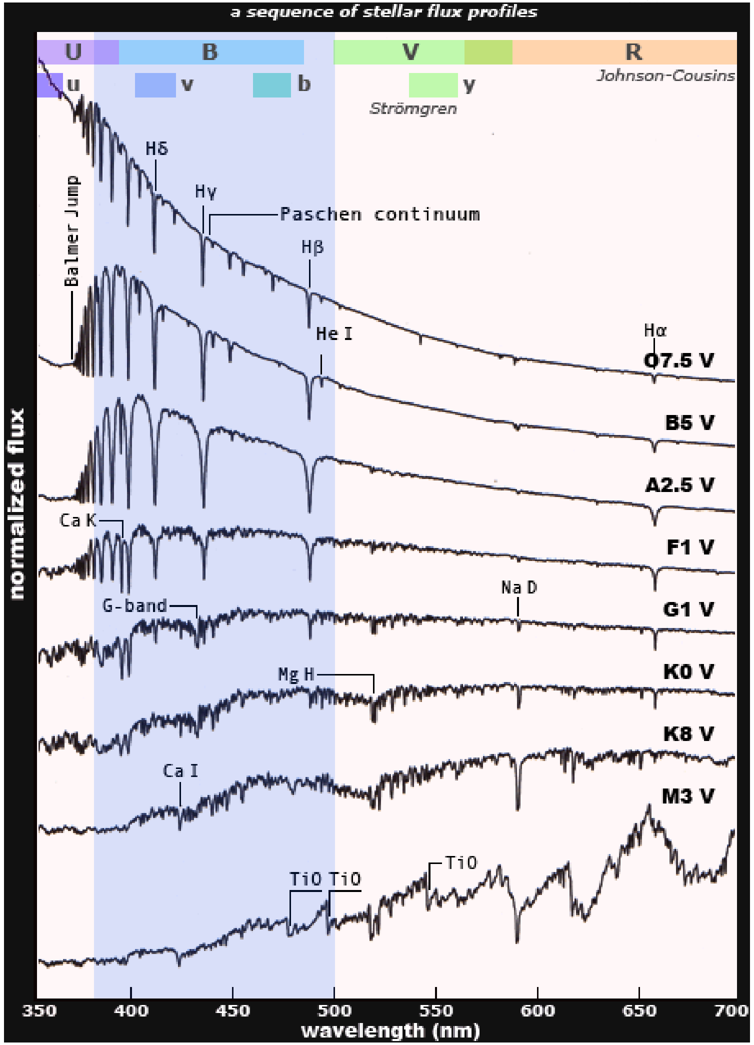
Spectral Classes



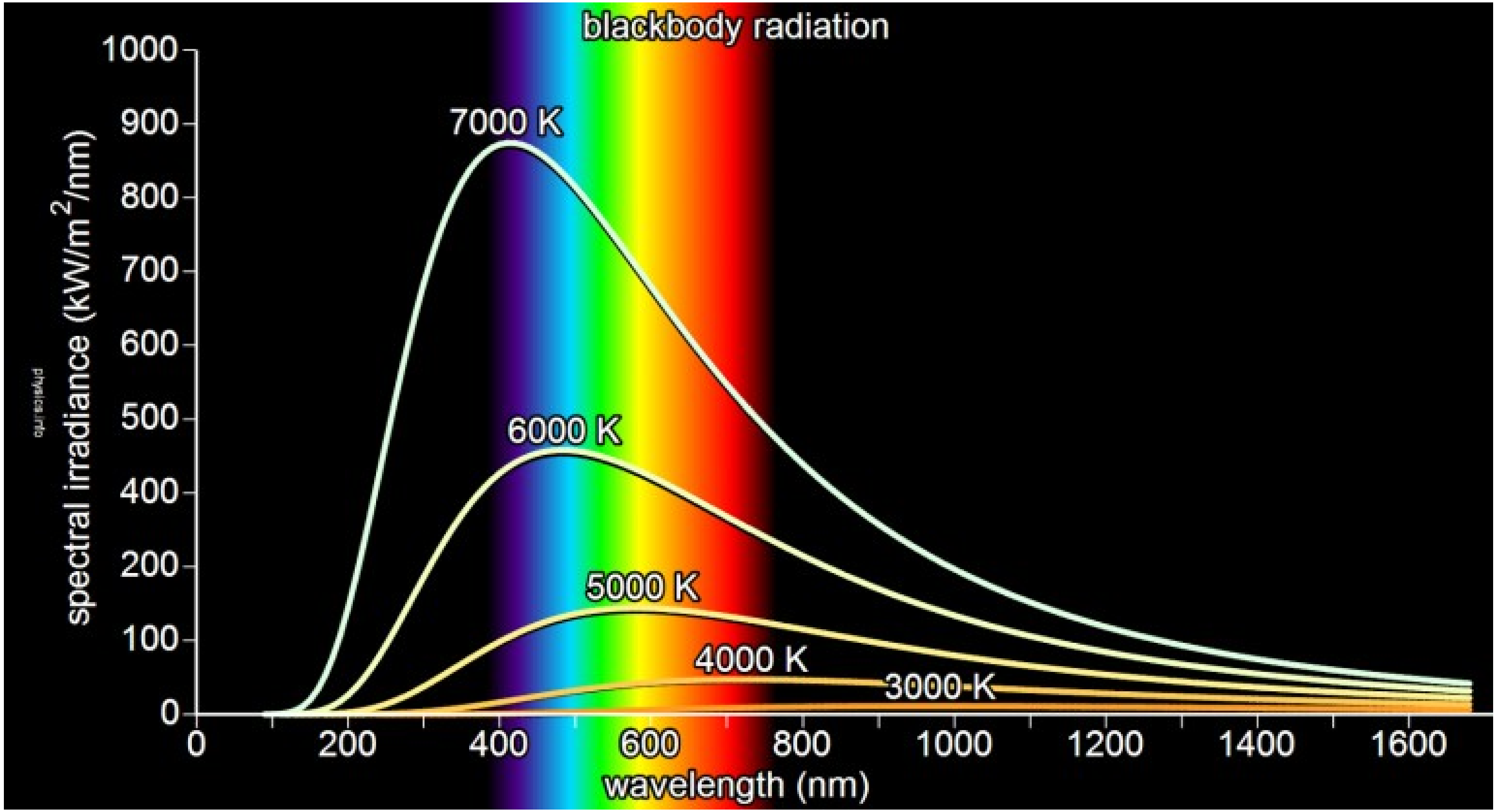
Based on their spectra, stars can be classified in 7 ***spectral classes*** or ***types***: O, B, A, F, G, K and M

Each class is further sub-divided in 10 sub-classes, from 0 to 9, e.g. A0, F5, M1 etc. Our Sun is a G2 star.

Spectral classes are associated with ***temperature*** and ***color***.



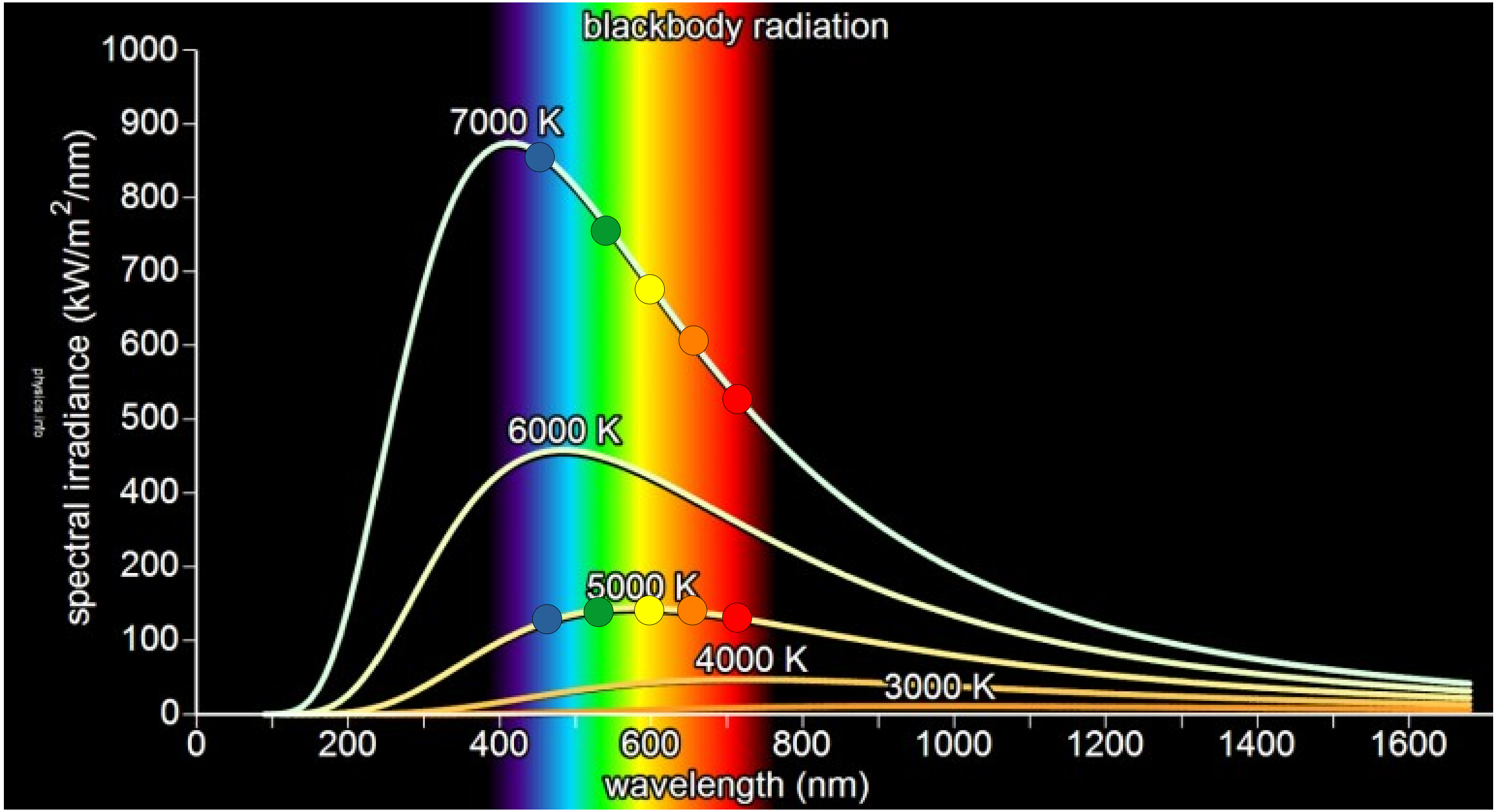
Focusing on temperature



Planck's Law shows us that the amount of energy emitted at a frequency range (or wavelength range) depends strongly on the temperature.

If we could adjust our observations and only target specific ranges, we could then obtain information on the temperature!

Filters and the SED



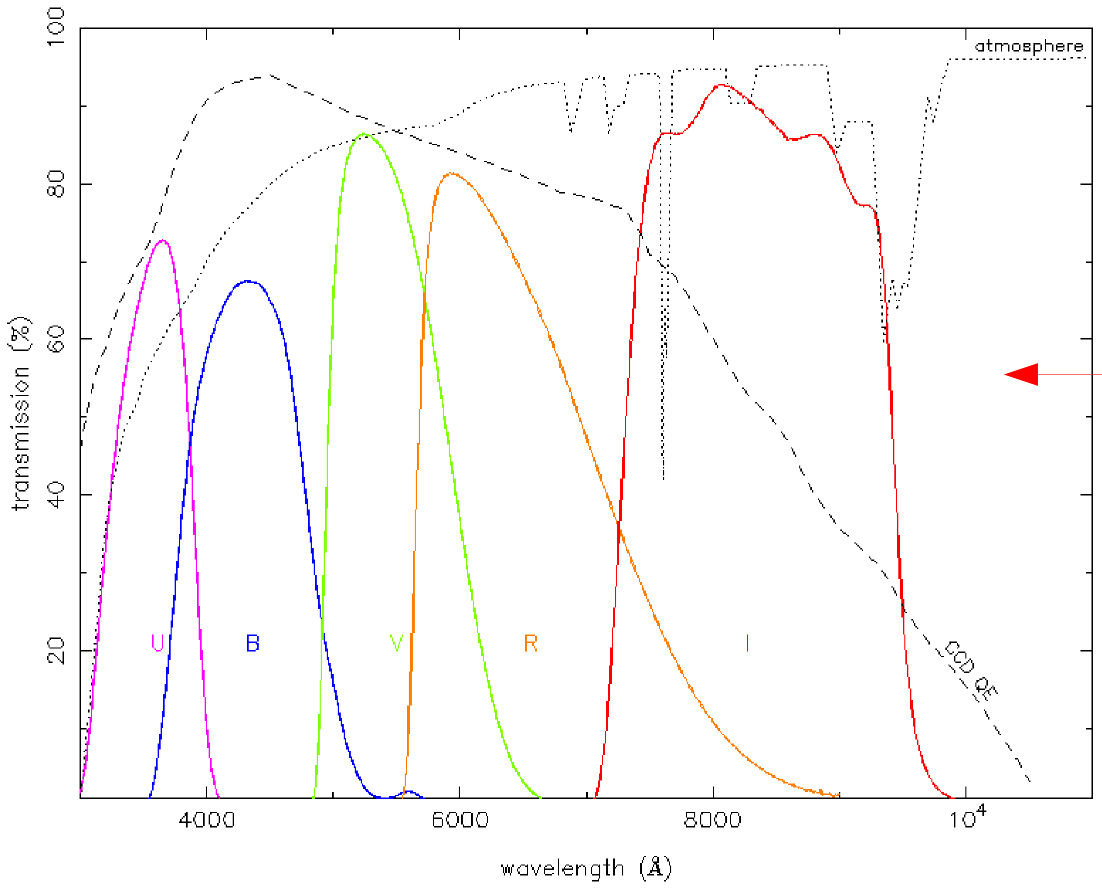
Using filters we can pick and choose which part of the spectrum we are observing.

Thus, we start sampling the so called:

Spectral
Energy
Distribution

(**SED**) of objects.

Filters – the Johnson UBVRI system

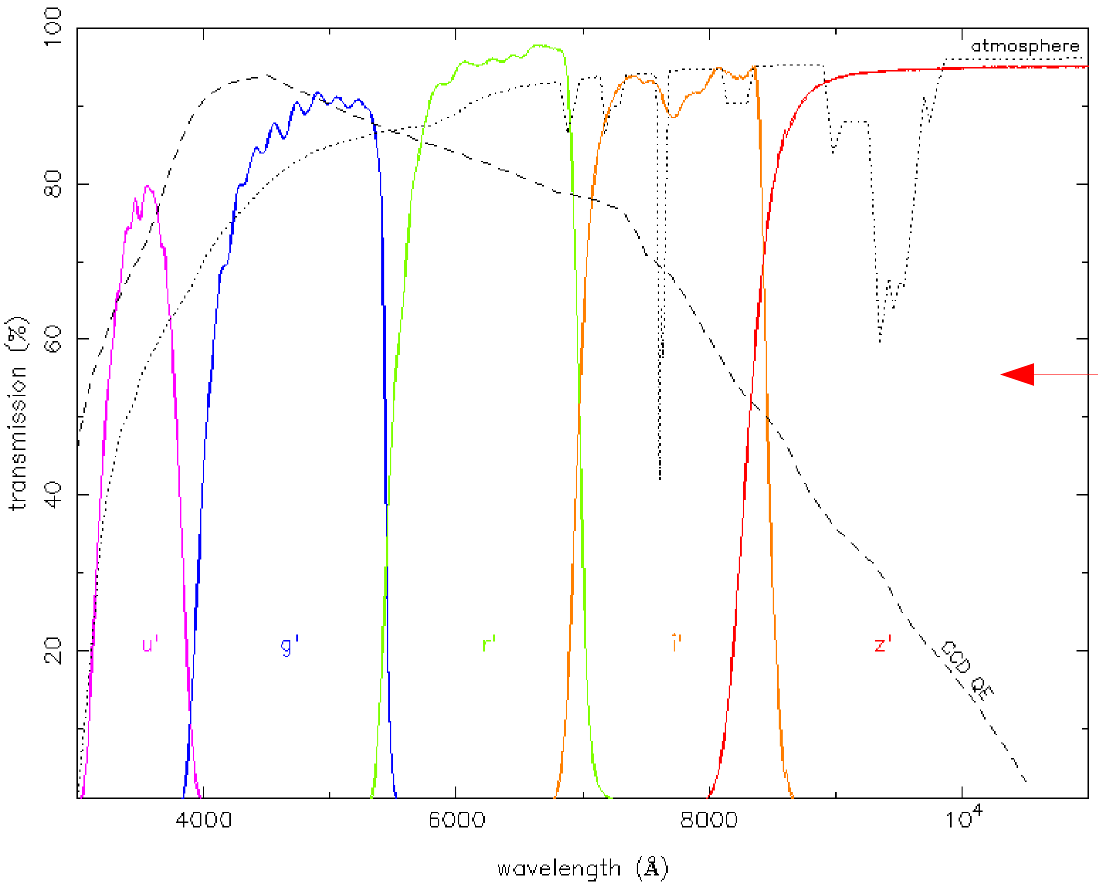


We can isolate parts of the optical spectrum and only record the light in these specific regions by using ***astronomical filters***.

The **Johnson** filter system UBVRI, was the most widely used system in astronomy (until the early 2000's)

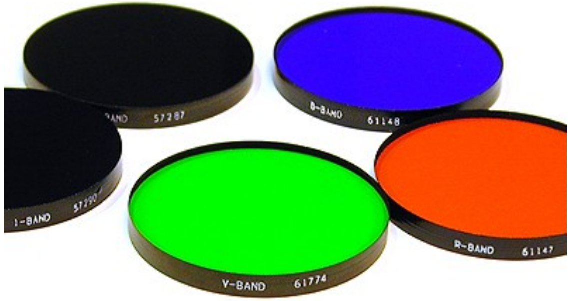
Originally developed by Johnson and Morgan (UBV), with additions from Cousins (RI), redesigned by Bessel and Cron to better match the response of CCDs.

Filters – the SDSS ugriz system



The **S**loan **D**igital **S**ky **S**urvey (SDSS) filter system ugriz, has practically replaced the Johnson system in current-day astronomy.

Color Index



A set of Johnson filters

A set of SDSS filters

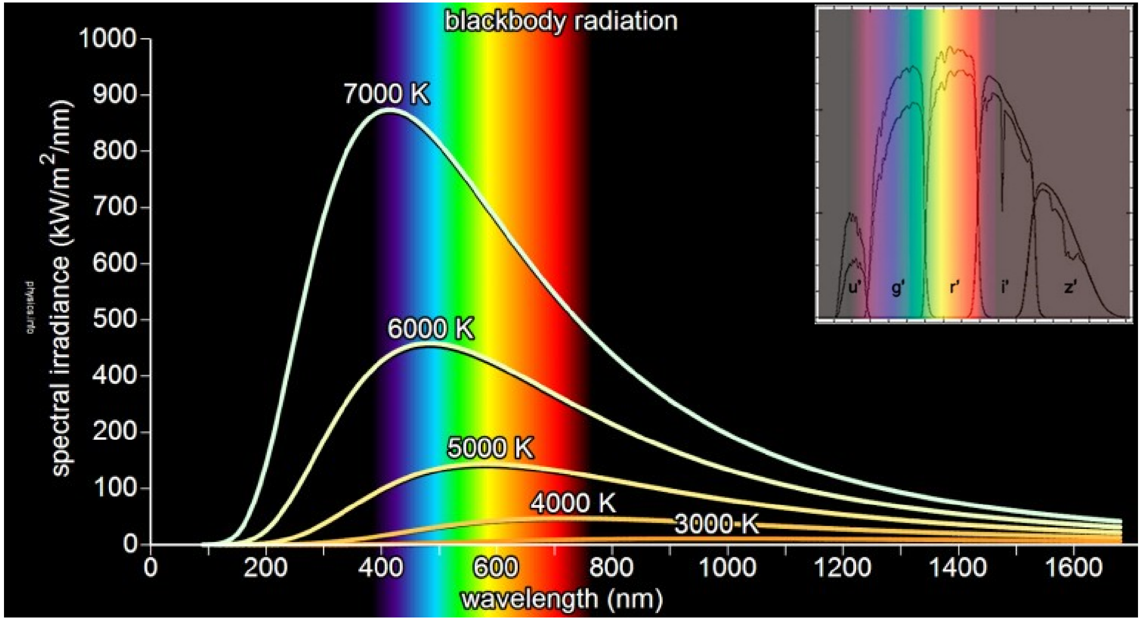
By observing in different filters, we can obtain magnitudes (apparent or absolute) in the given filter.

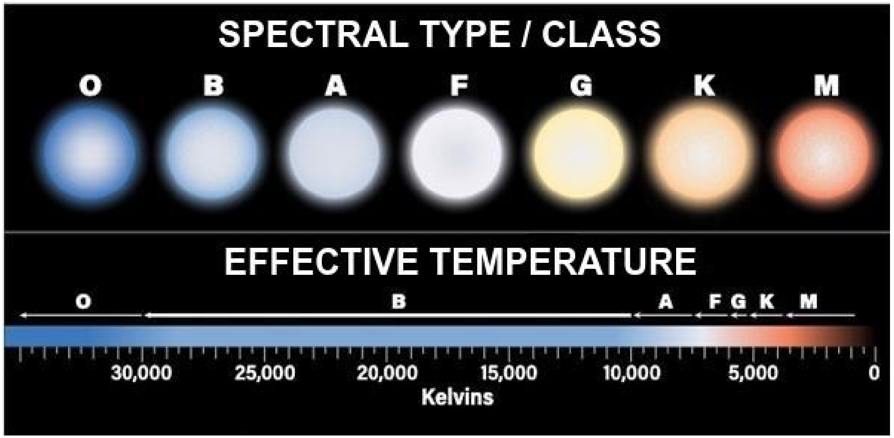
These are indicated either by a *subscript,* e.g.
$\mathbf{m_V}$ – for apparent magnitude in Johnson V
$\mathbf{M_g}$ – for absolute magnitude in SDSS g

or by simply using the *filter name, e.g.*
B = 15 – for a star of magnitude 15 in the Johnson B-filter
r = 19 – for a star of magnitude 19 in the SDSS r-filter

The difference between two magnitudes obtained in two different filters is called ***color index***:
B-V = $m_B - m_V$ ⟶ the magnitude difference in B & V
g-r = $m_g - m_r$ ⟶ the magnitude difference in g & r

Combining all the information





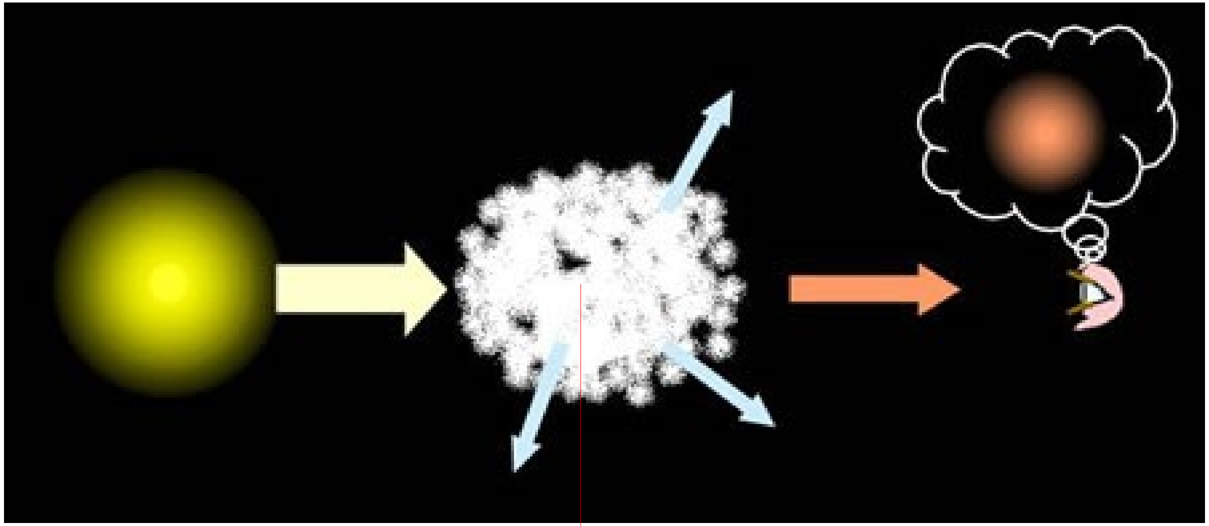
Some examples:

SpT A5: g-r ~ 0
SpT G5: g-r ~ 0.5
SpT M0: g-r ~ 1.3

Hot star → **high T$_{eff}$** → **blue** color → $F_g > F_r \Leftrightarrow m_g < m_r \Leftrightarrow g - r < 0$

Cool star → **low T$_{eff}$** → **red** color → $F_g < F_r \Leftrightarrow m_g > m_r \Leftrightarrow g - r > 0$

Interstellar extinction and reddening



$g_0 = 14.5$
$r_0 = 14.0$
$\Big\} \; g_0 - r_0 = 0.5$

$g = 16.0$
$r = 14.7$
$\Big\} \; g - r = 1.3$

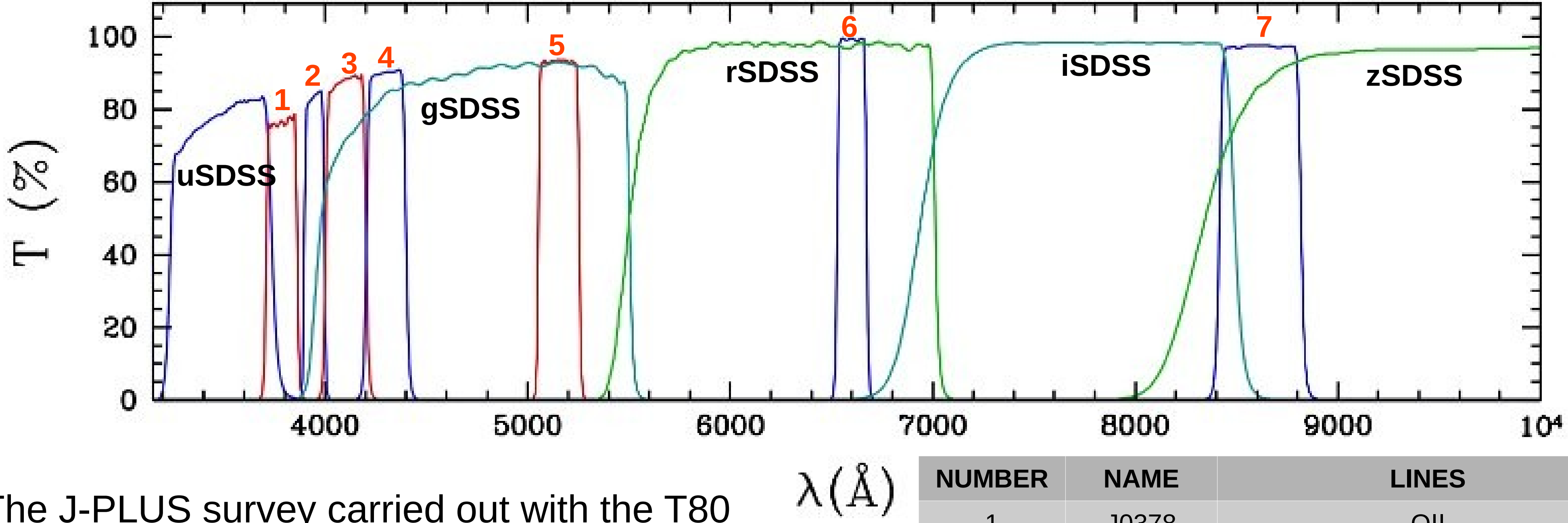
Light coming to us from distant objects can be affected by ***interstellar*** material, like dust grains.

Dust grains will absorb and scatter the light; blue wavelenghts more than red.

As a result stars appear ***dimmer*** and ***redder*** than they really are.

The quantity $E(g-r) = (g-r) - (g_0-r_0)$ is called ***color excess***.

The J-PLUS filter system

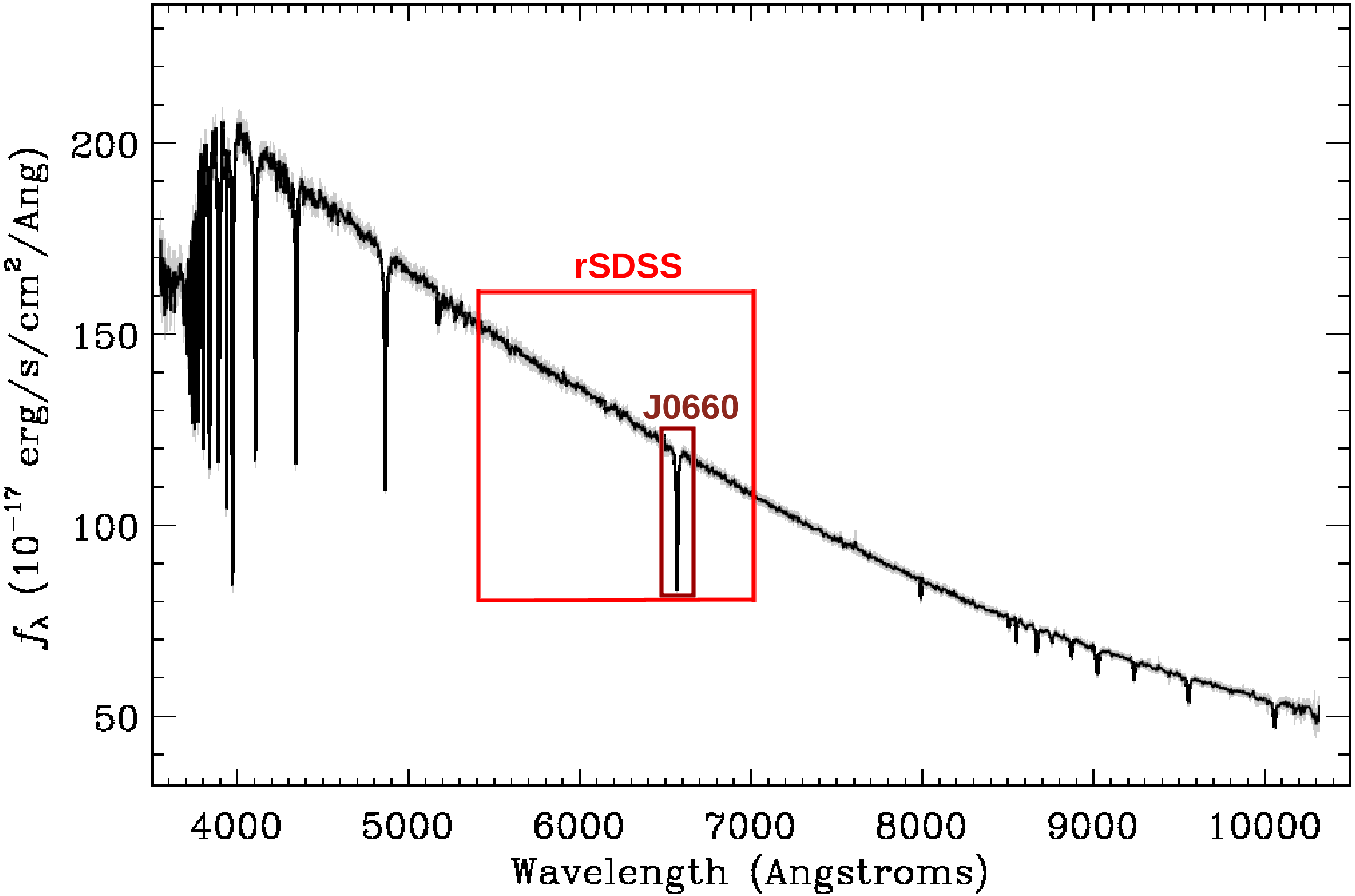


The J-PLUS survey carried out with the T80 telescope at the OAJ uses a 12 filter system.

Apart from the standard **SDSS** filters, **ugriz**, J-PLUS additionally uses **7 *narrowband*** filters, centered on specific spectral lines.

NUMBER	NAME	LINES
1	J0378	OII
2	J0395	Ca H+K
3	J0410	Hδ
4	J0430	G-band
5	J0515	Mgb triplet, OIII
6	J0660	Hα
7	J0861	Ca triplet

Narrowband photometry

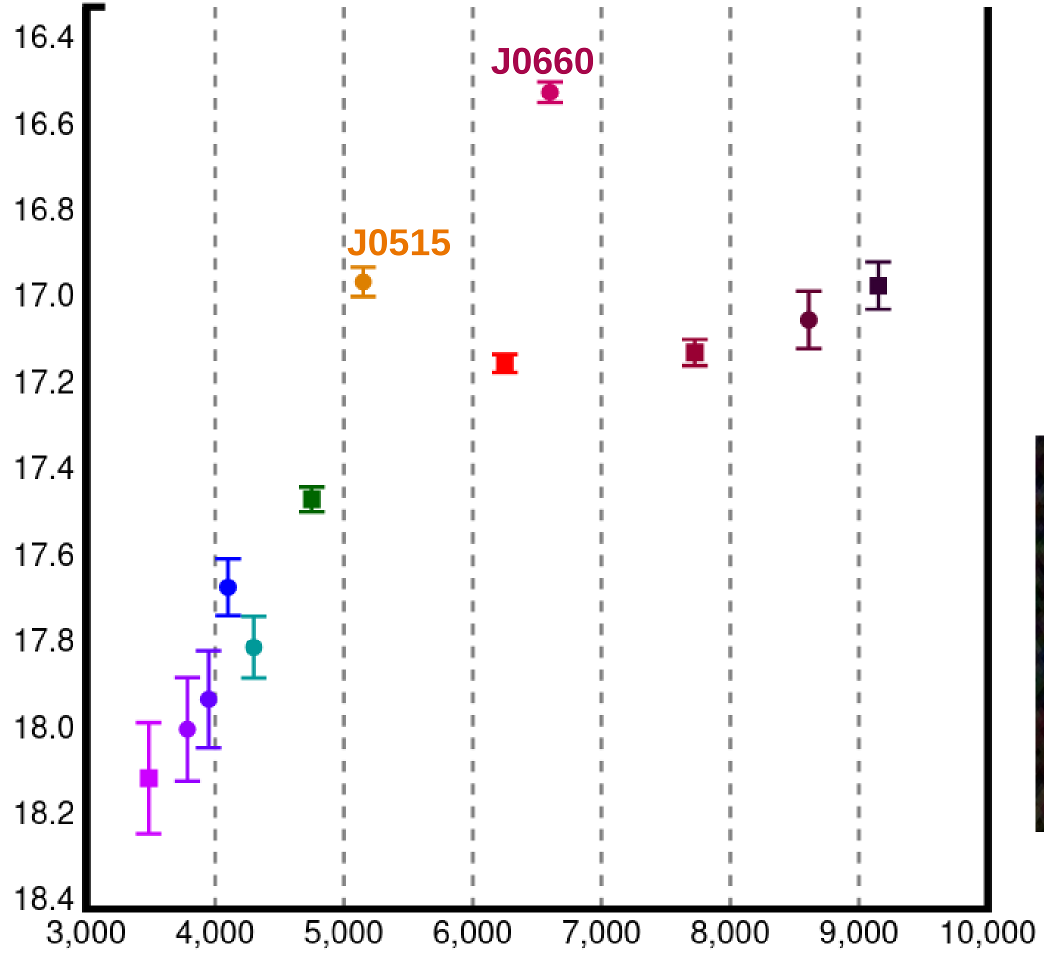


The flux measured in the **rSDSS** filter will be mainly dominated by the continuum.

In comparison to this flux, the flux in **J0660** will show a deficit (or excess) when the Hα line is seen in absorption (or emission).

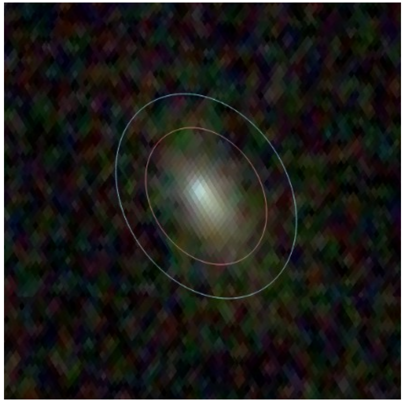
This will translate into an observable difference in the magnitudes in $\mathbf{m_{rSDSS}}$ and $\mathbf{m_{J0660}}$.

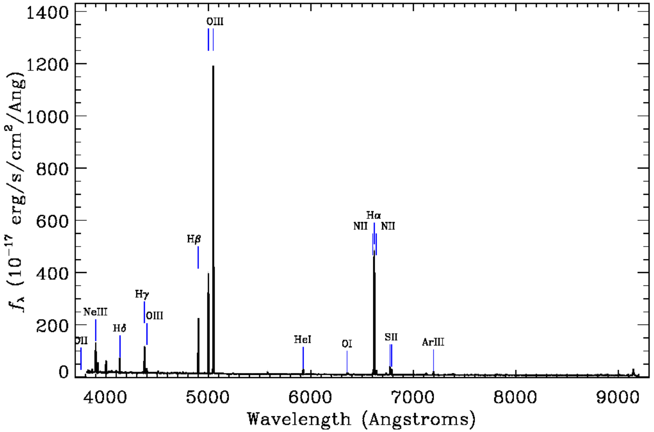
Narrowband photometry



Example of an object showing clear excess in the narrowband photometry of the **J0515** and **J0660** filters.

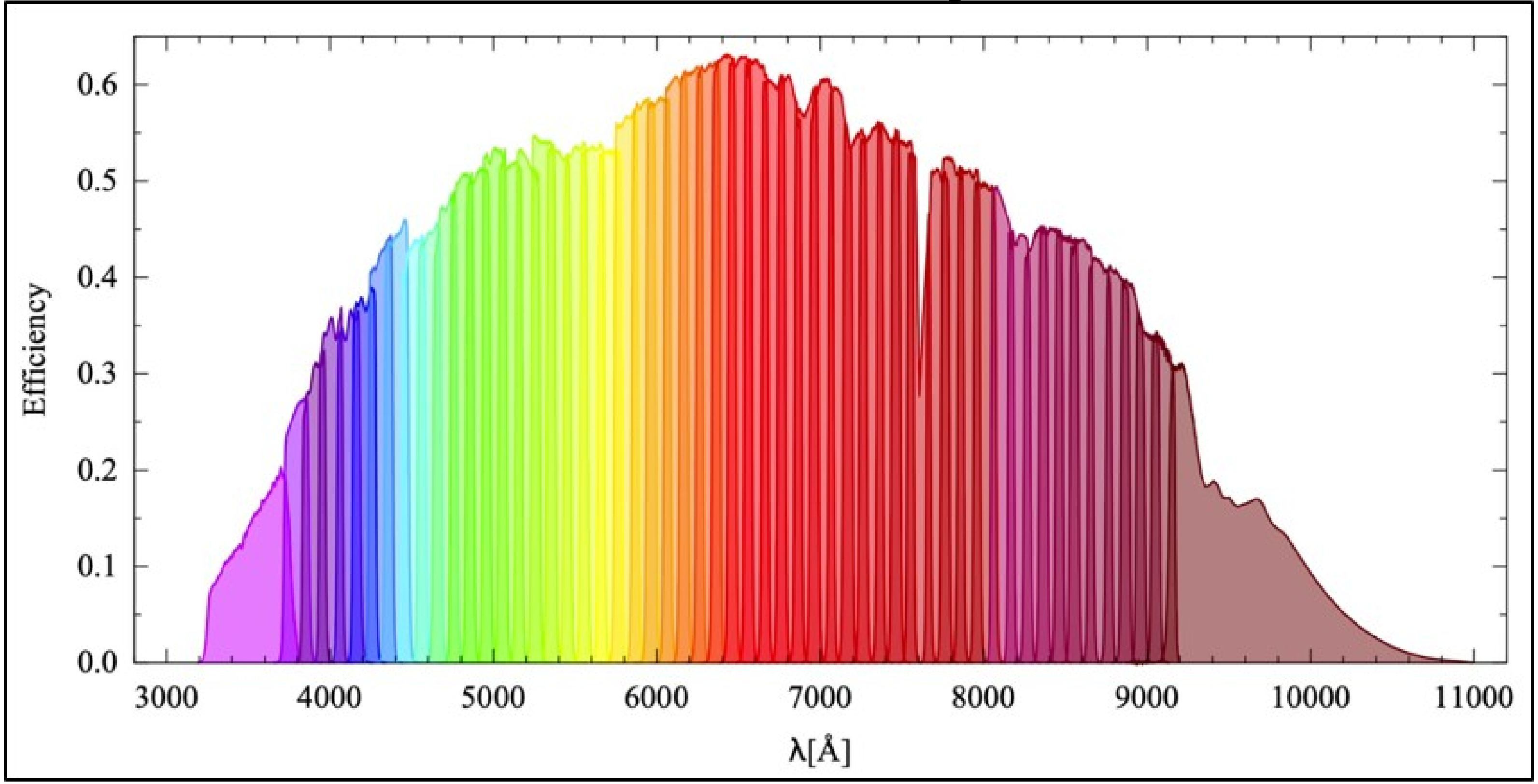
This object is an *extreme-emission line galaxy* identified by our CEFCA colleague Alejandro Lumbreras-Calle (2022, A&A, 668).





Detailed spectrum

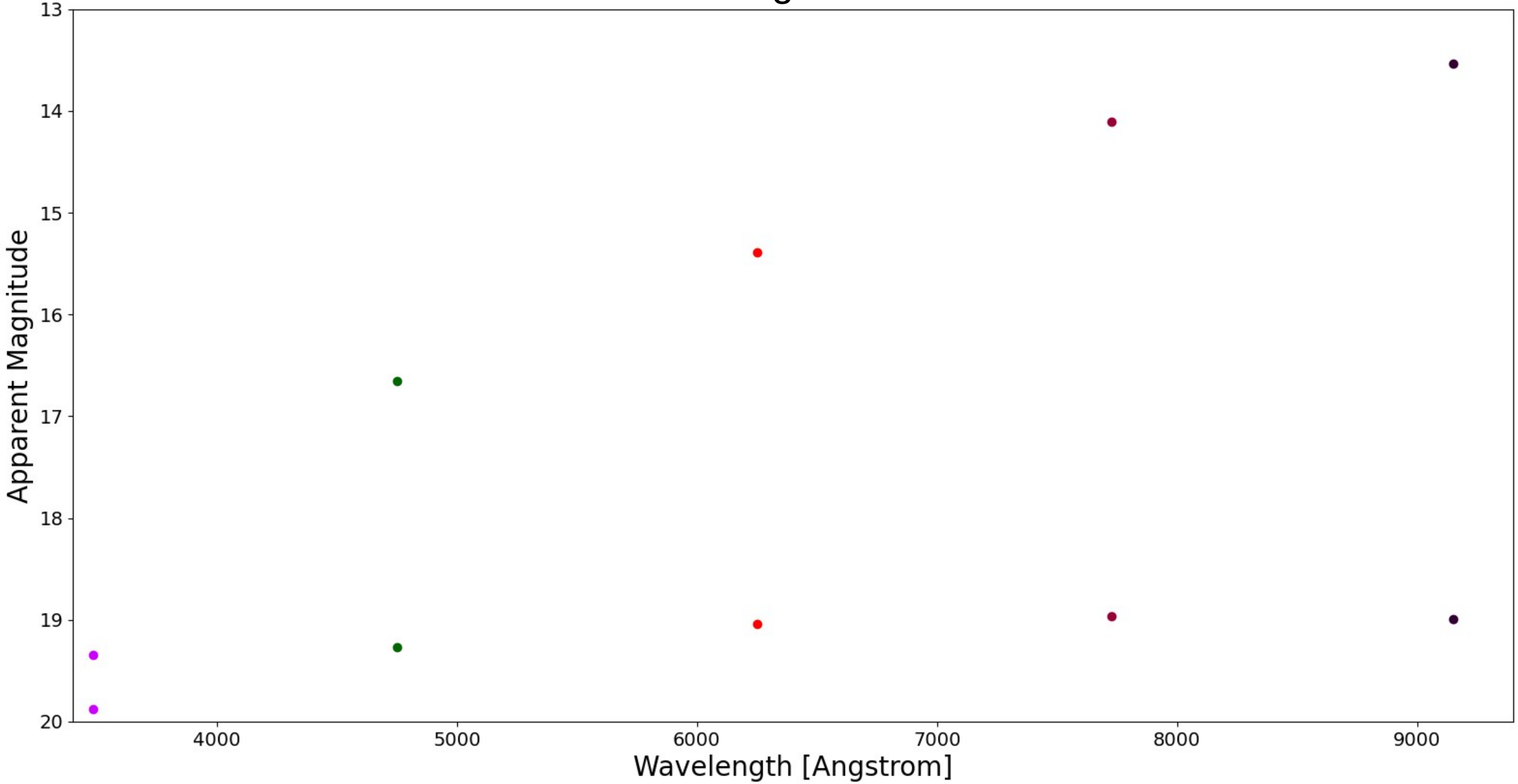
The J-PAS filter system



The J-PAS survey carried out with the T250 telescope at the OAJ uses an innovative filter system consisting of ***56 narrowband*** filters, convering the entire optical range. The *Spectral Energy Distributions* (**SEDs**) are sometimes referred to as ***photospectra***.

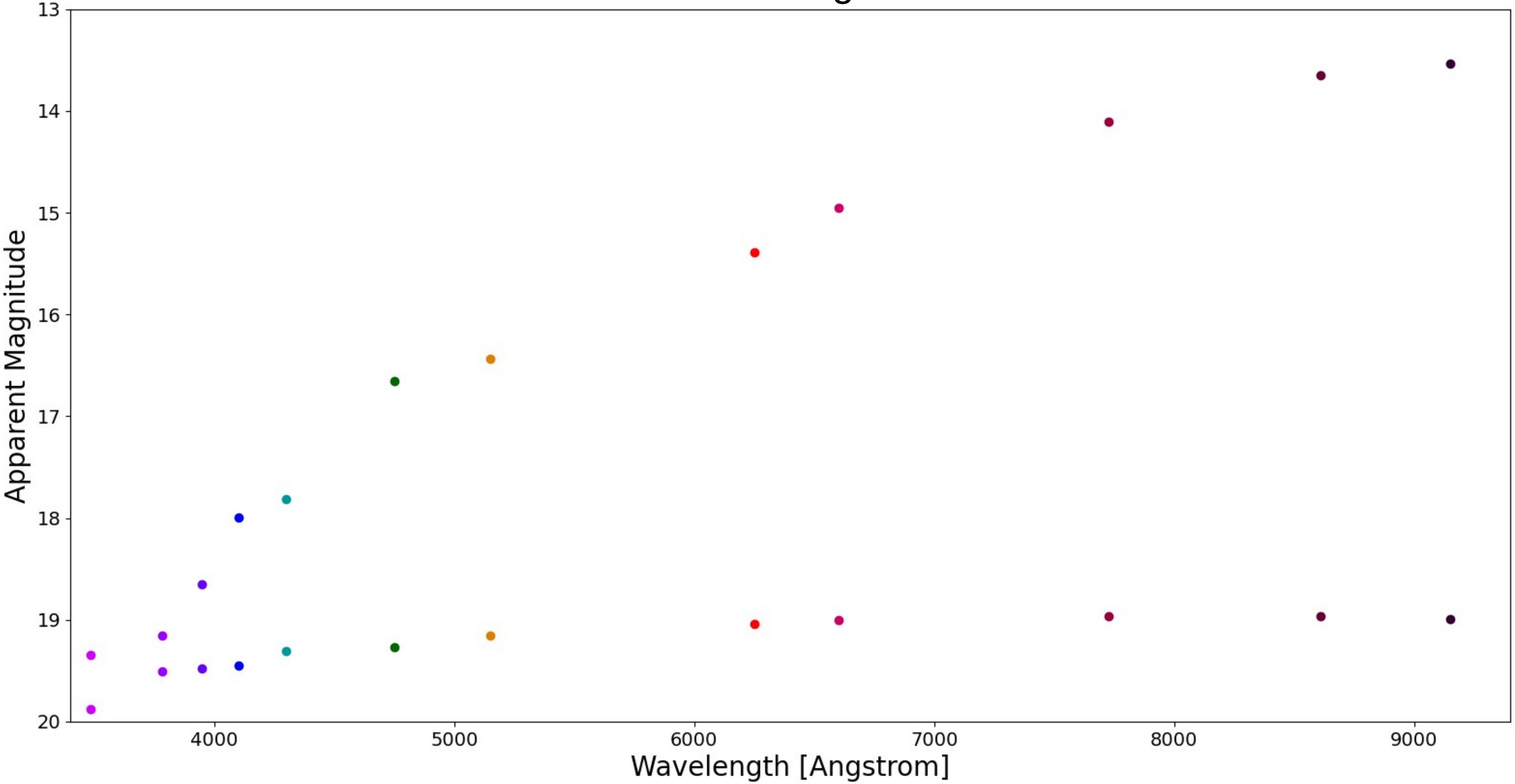
Spectral Energy Distribution (SED)

Two stars observed using the 5 filters of SDSS



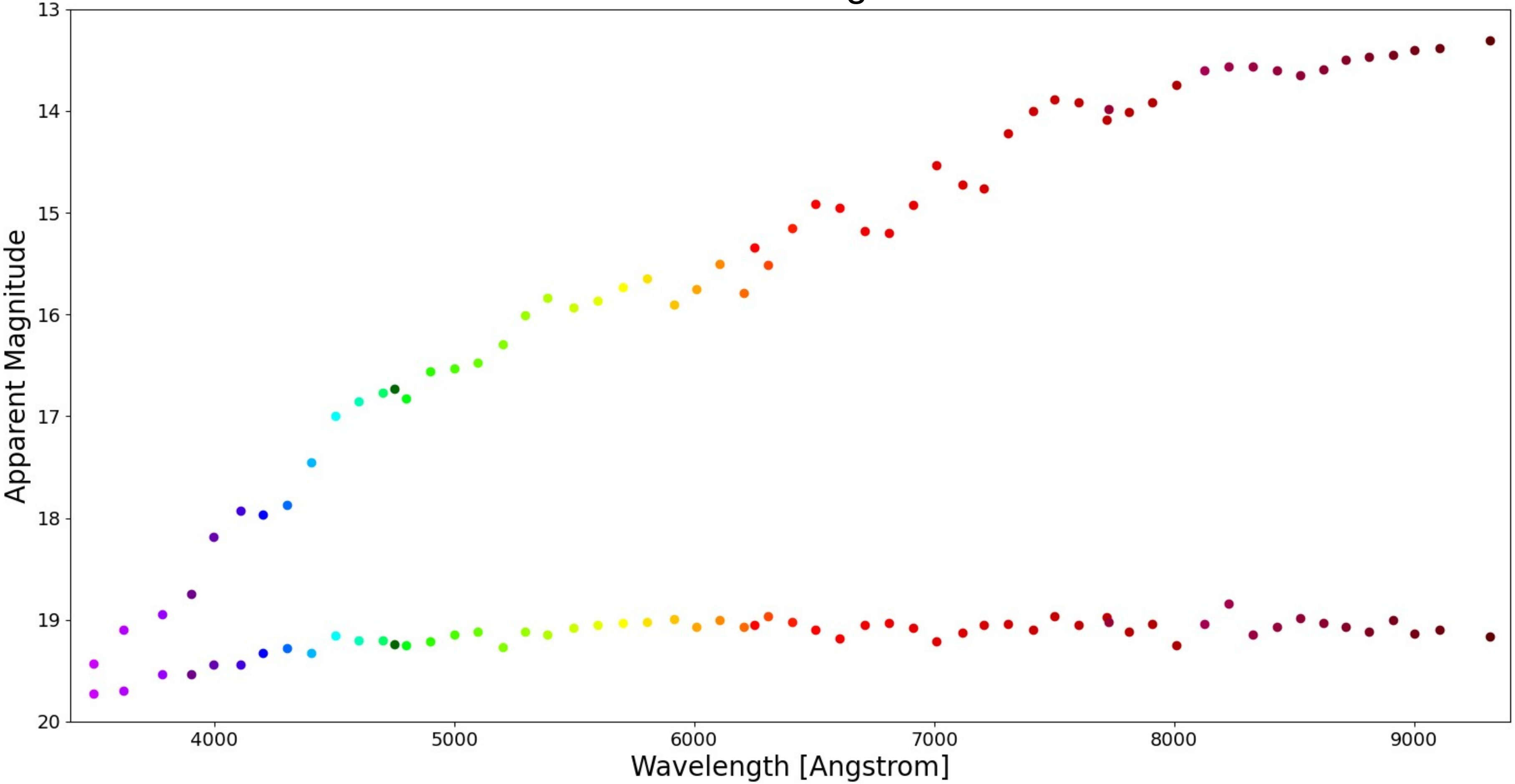
Spectral Energy Distribution (SED)

The same two stars observed using the 12 filters of J-PLUS



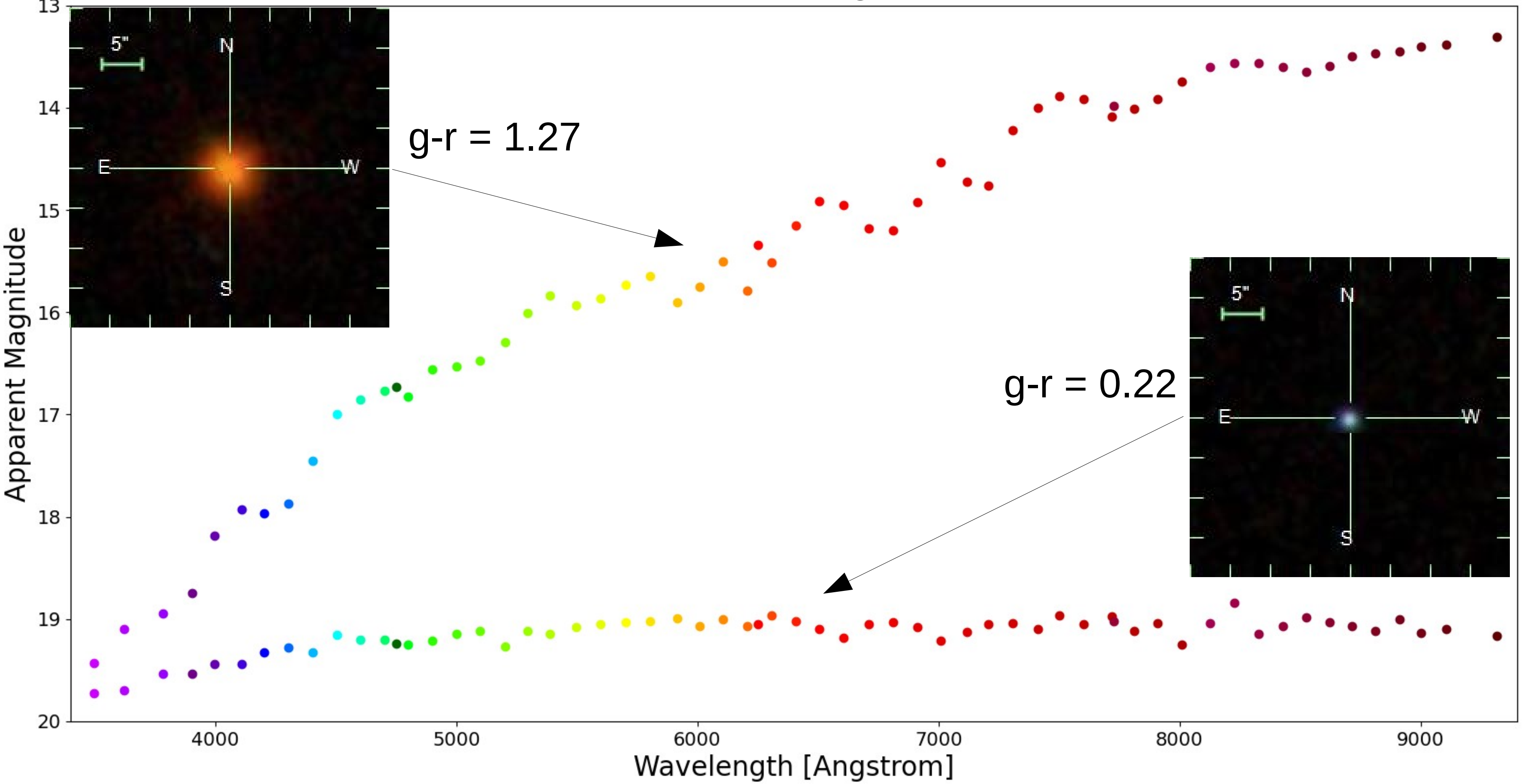
Spectral Energy Distribution (SED)

The same two stars observed using the 56 filters of J-PAS



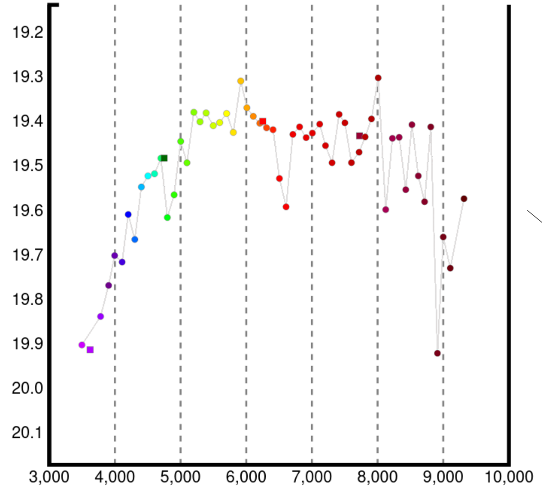
Spectral Energy Distribution (SED)

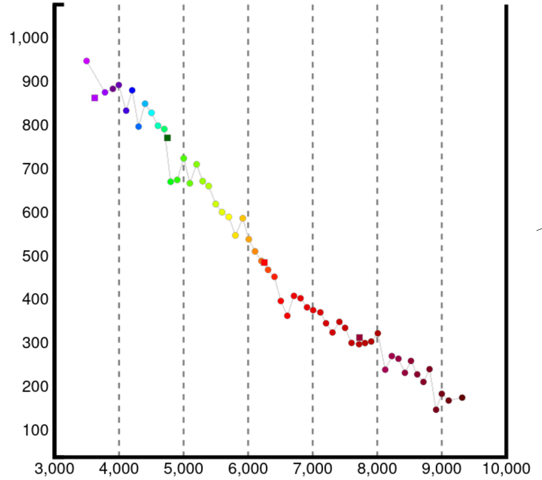
The same two stars observed using the 56 filters of J-PAS



Why photometry?

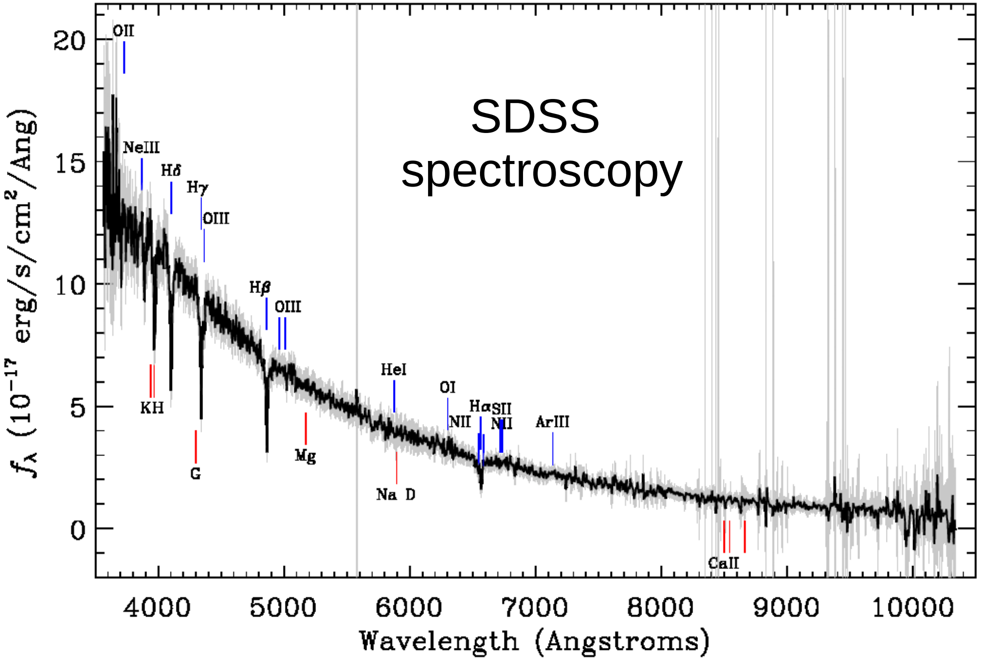
J-PAS 56 filter photometry





While photometry cannot compete with spectroscopy in terms of resolution and information, it is *a lot* faster.

The goal is to be able to select interesting objects from all-sky photometry; the photometry needs to be good enough for that!

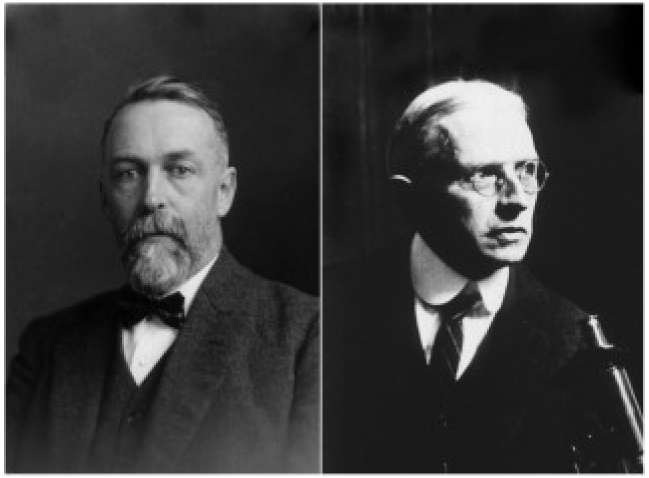


Doing

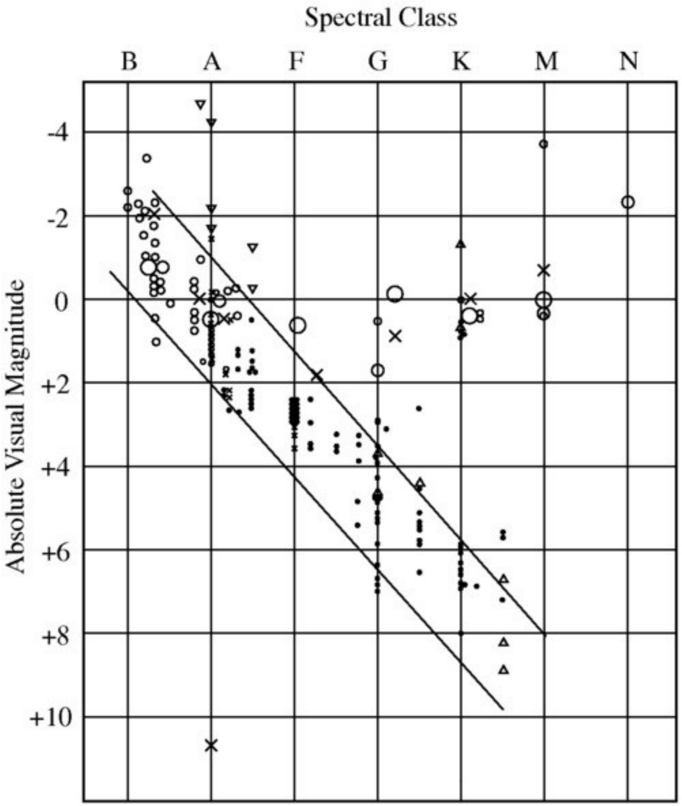
Science

The Hertzsprung-Russell diagram (H-R)

- Ejnar Hertzsprung (1911): Luminosity as a function of color
- Henry Norris Russell (1913): Luminosity as a function of SpT
- The H-R diagram allowed astronomers to develop the theory of stellar evolution.
- It is a visual representation of “everything” we know about stars!



Ejnar Hertzsprung   Henry Norris Russell



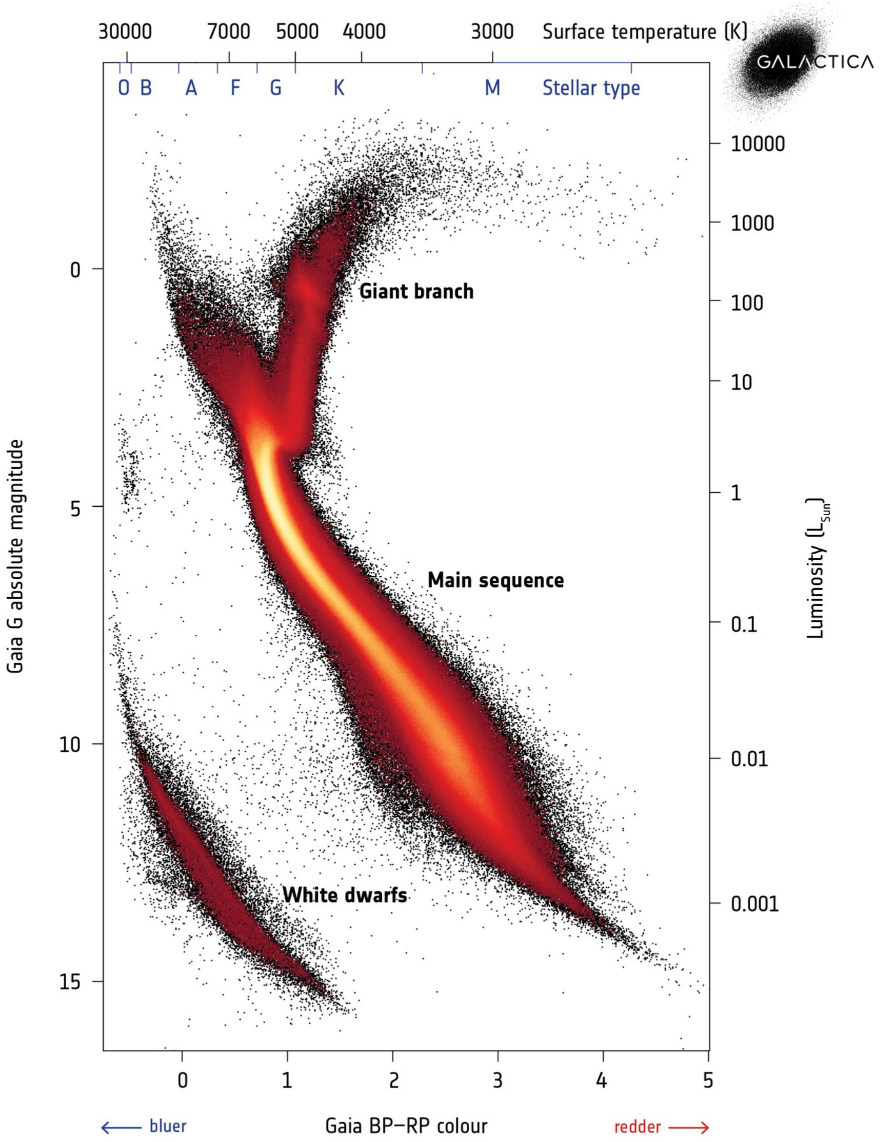
The original diagram from Russell

The H-R diagram from Gaia

The Gaia satellite has provided astronomers with the most detailed H-R diagram ever created, containing close to 2 billion stars!

Gaia uses two band-passes, the *blue* **BP** and the *red* **RP** to obtain colors.

It also uses a filterless band, the *Gaia* **G** to obtain absolute magnitudes (remember that Gaia's main job is to measure parallaxes...)



The H-R diagram from Gaia

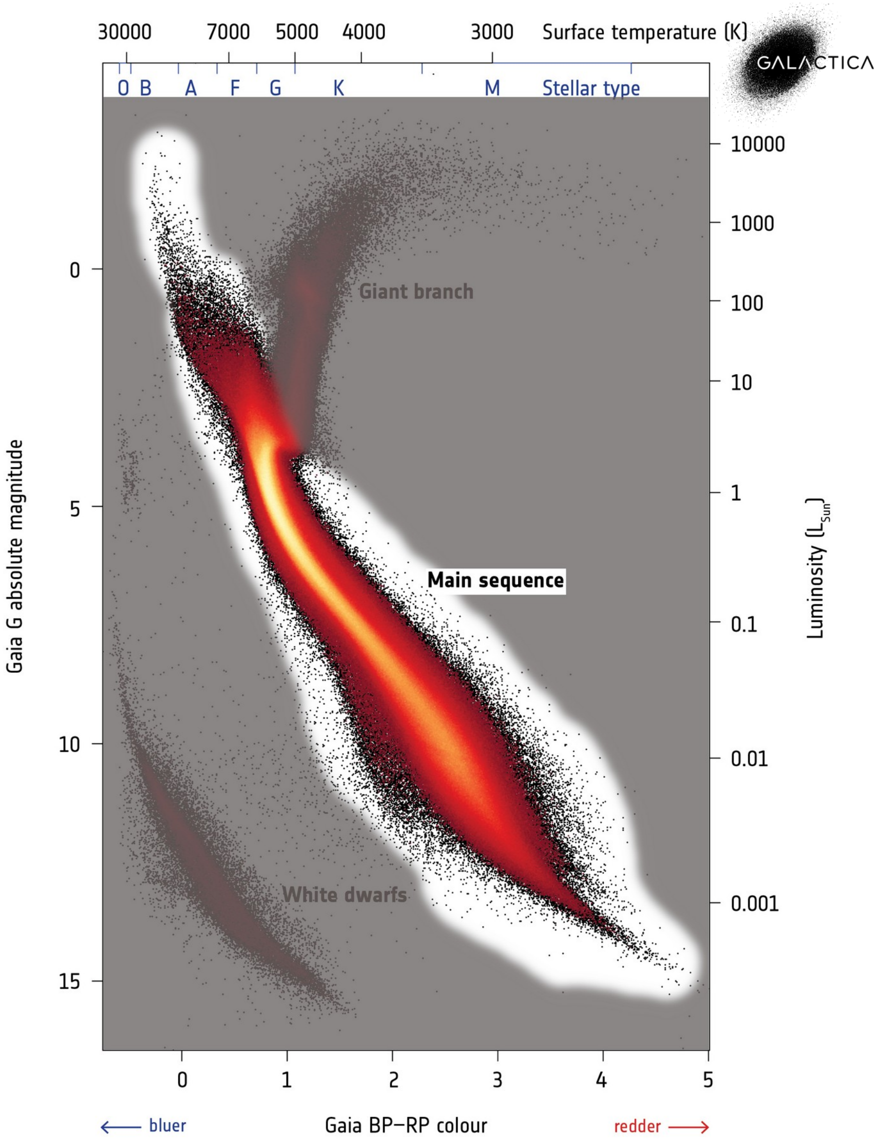
Most stars occupy a region of the H-R diagram known as ***the Main Sequence (MS)***.

Stars spend most of their lives on the MS. In this phase they sustain steady nuclear fusion reactions in their cores, i.e. converting *Hydrogen* into *Helium*.

Going in a direction from the bottom-right (**red** color) to the top-left (**blue** color):

- **Mass** : is increasing
- **Radius** : is increasing
- **Temperature**: is increasing
- **Luminosity** : is increasing

- **Life-time** : is *decreasing*

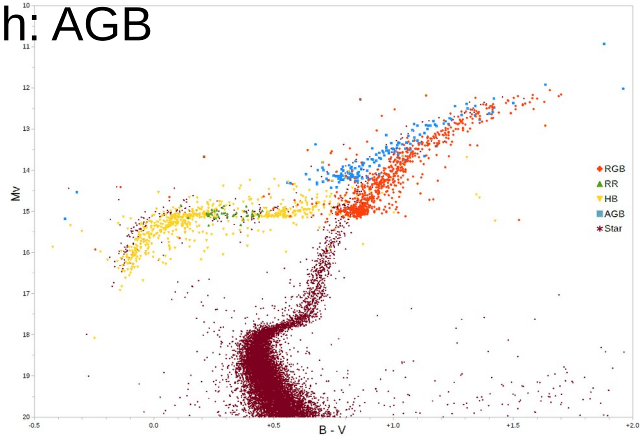


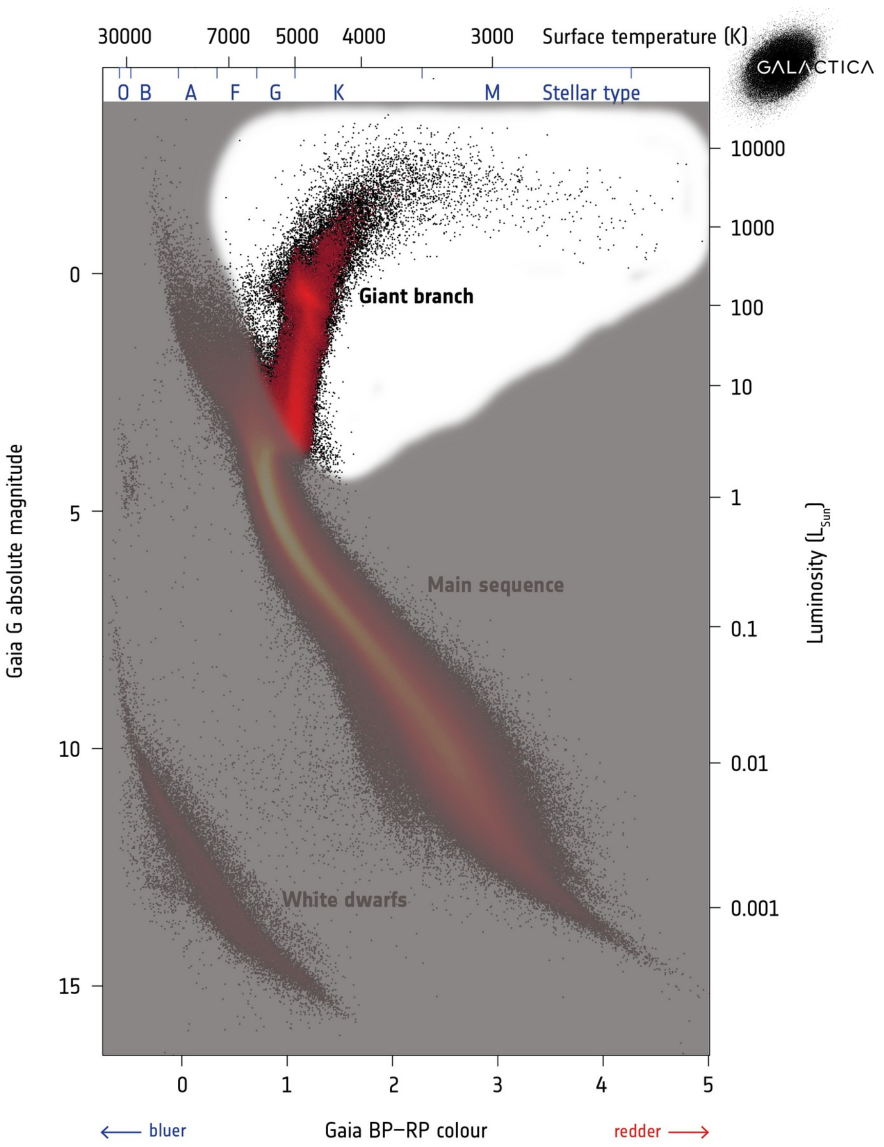
The H-R diagram from Gaia

When stars exhaust the hydrogen in their cores, they begin their *evolution*. They move away from the MS and ascend towards the ***giant branch***.

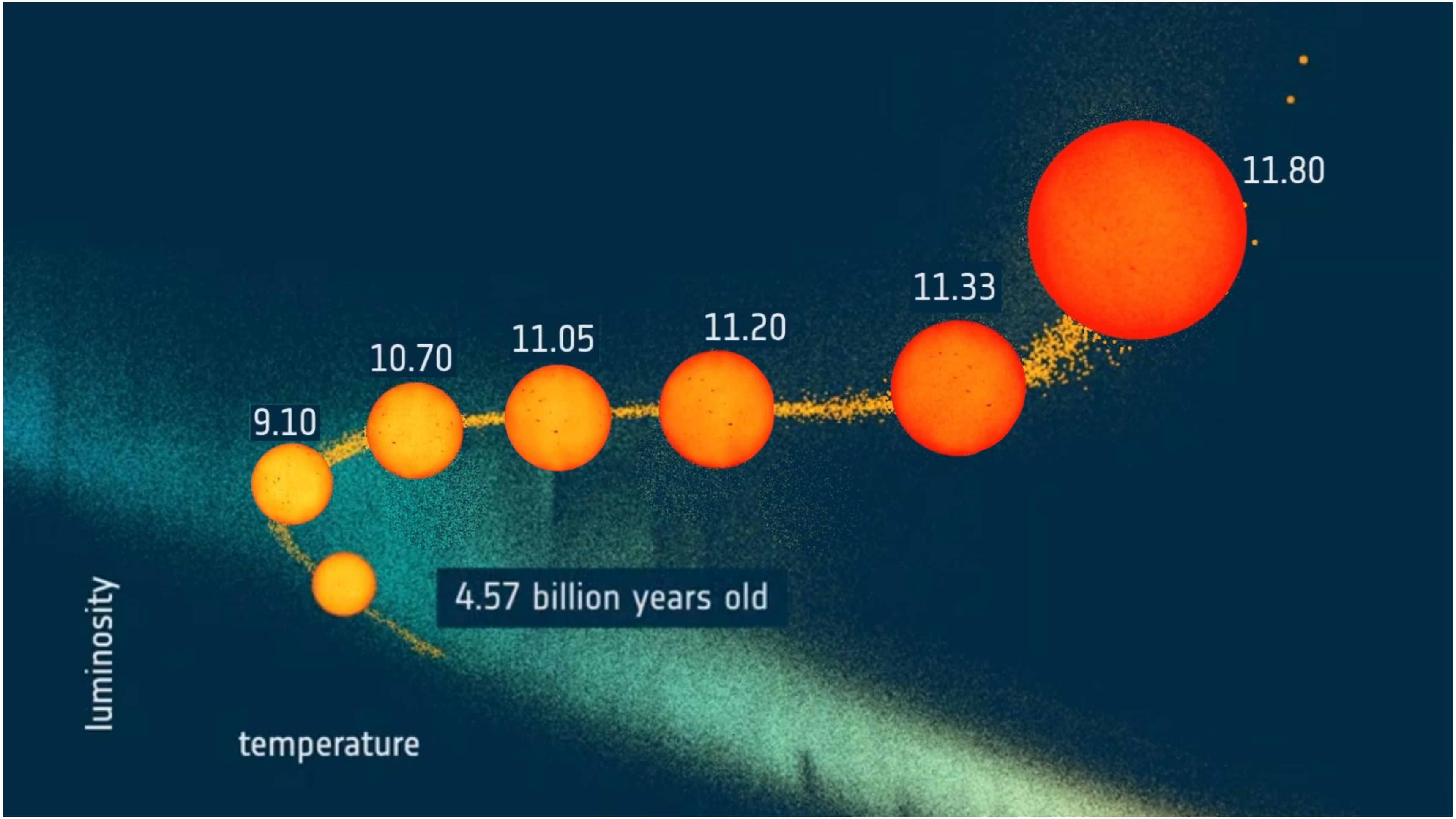
Depending on their mass, their metallicity and their internal structure, evolving stars can go through different channels/parts of the giant branch region:

Red Giant Branch: RGB
Horizontal Branch: HB
Asymptotic Giant Branch: AGB





The evolution track of our Sun



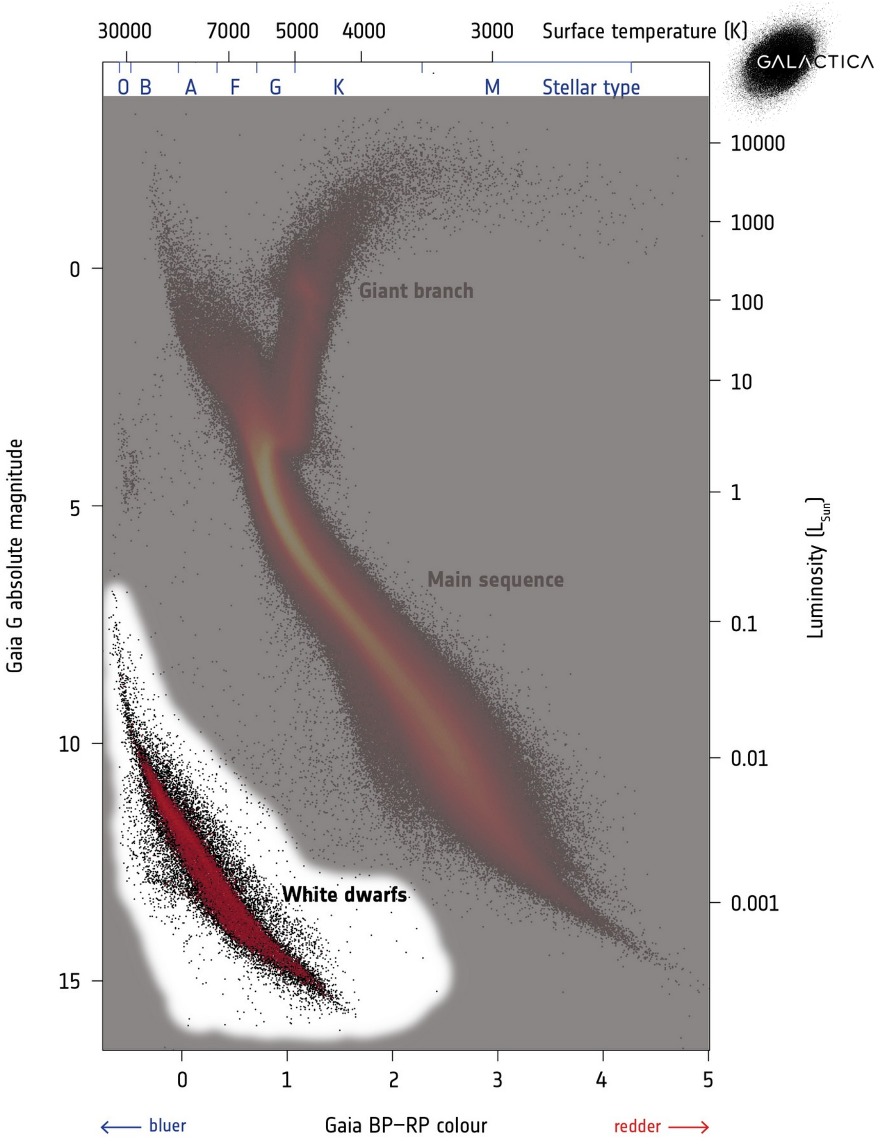
The H-R diagram from Gaia

Stars with (original) mass up to 8-10 solar masses (95% of the stars…) will end their evolution as a stellar remnant called a ***white dwarf (WD)***. Our own Sun will eventually become a white dwarf.

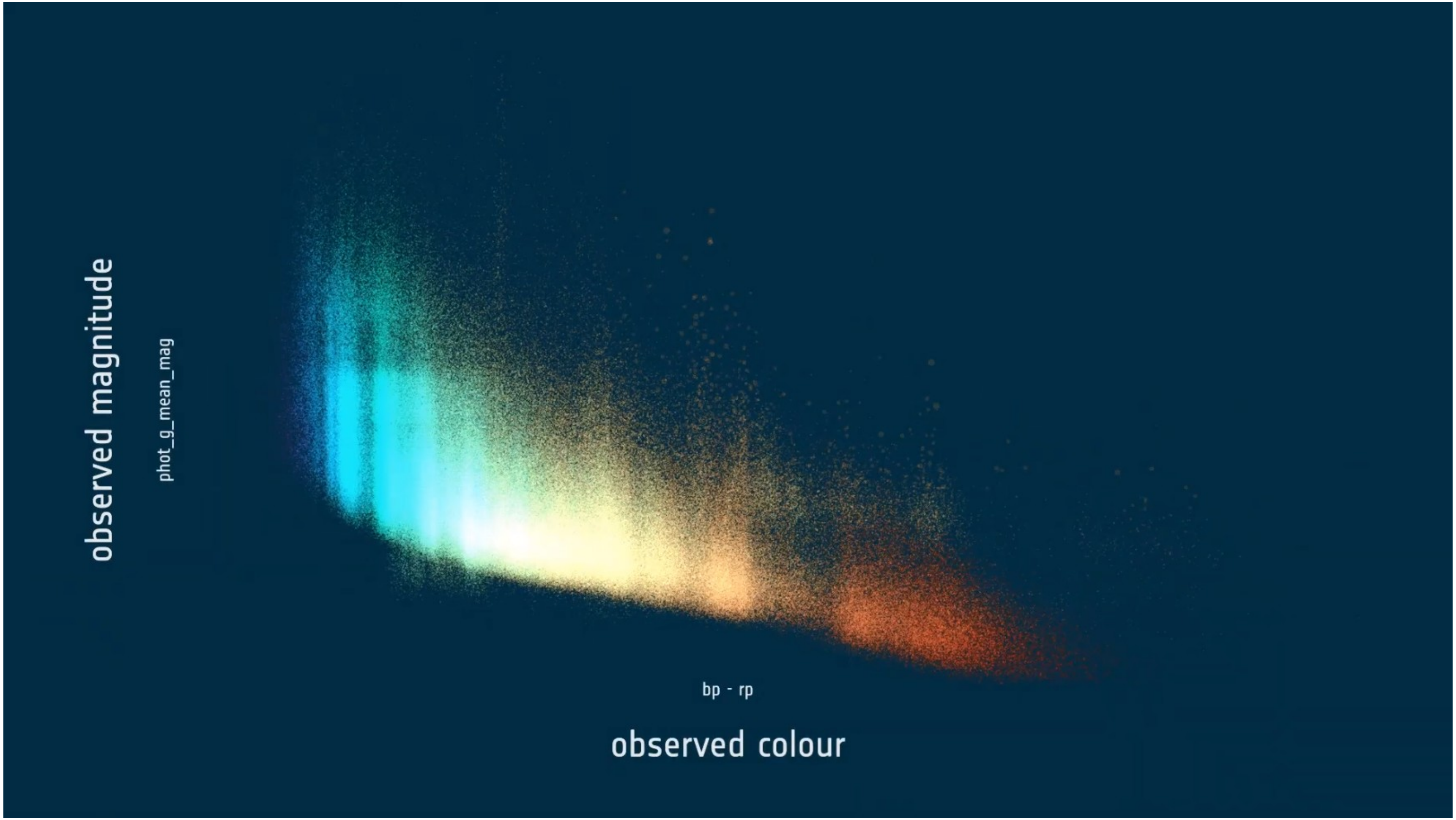
WDs no longer produce energy in their cores, they simply radiate away energy stored in them during the evolutionary process.

As a result of *only* radiating, they gradually cool down. With their *temperature* dropping, their *luminosity* is also dropping, and their color is becoming *redder*.

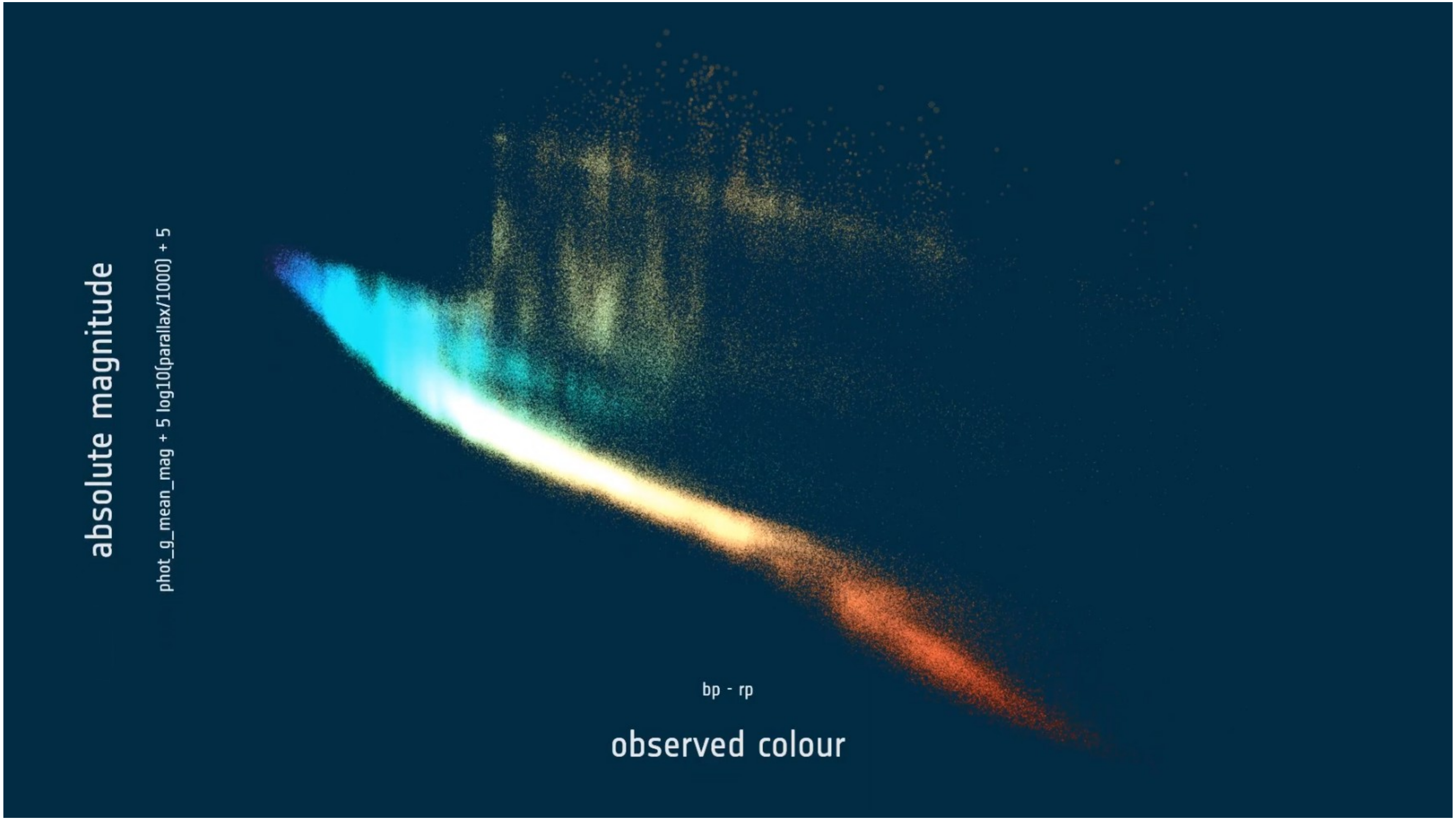
So WDs follow a top-left to bottom right track in the WD locus.



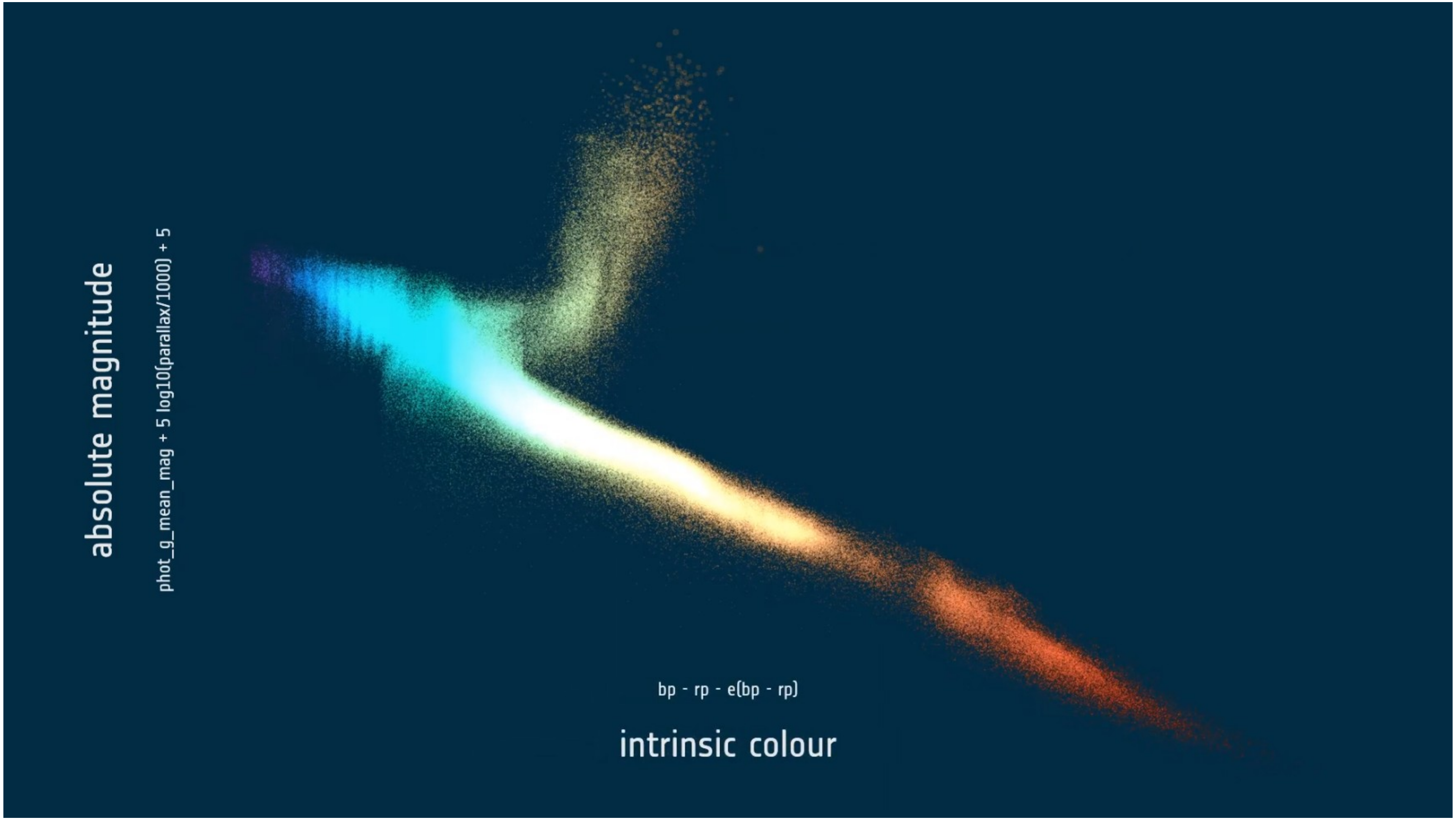
Constructing an H-R diagram



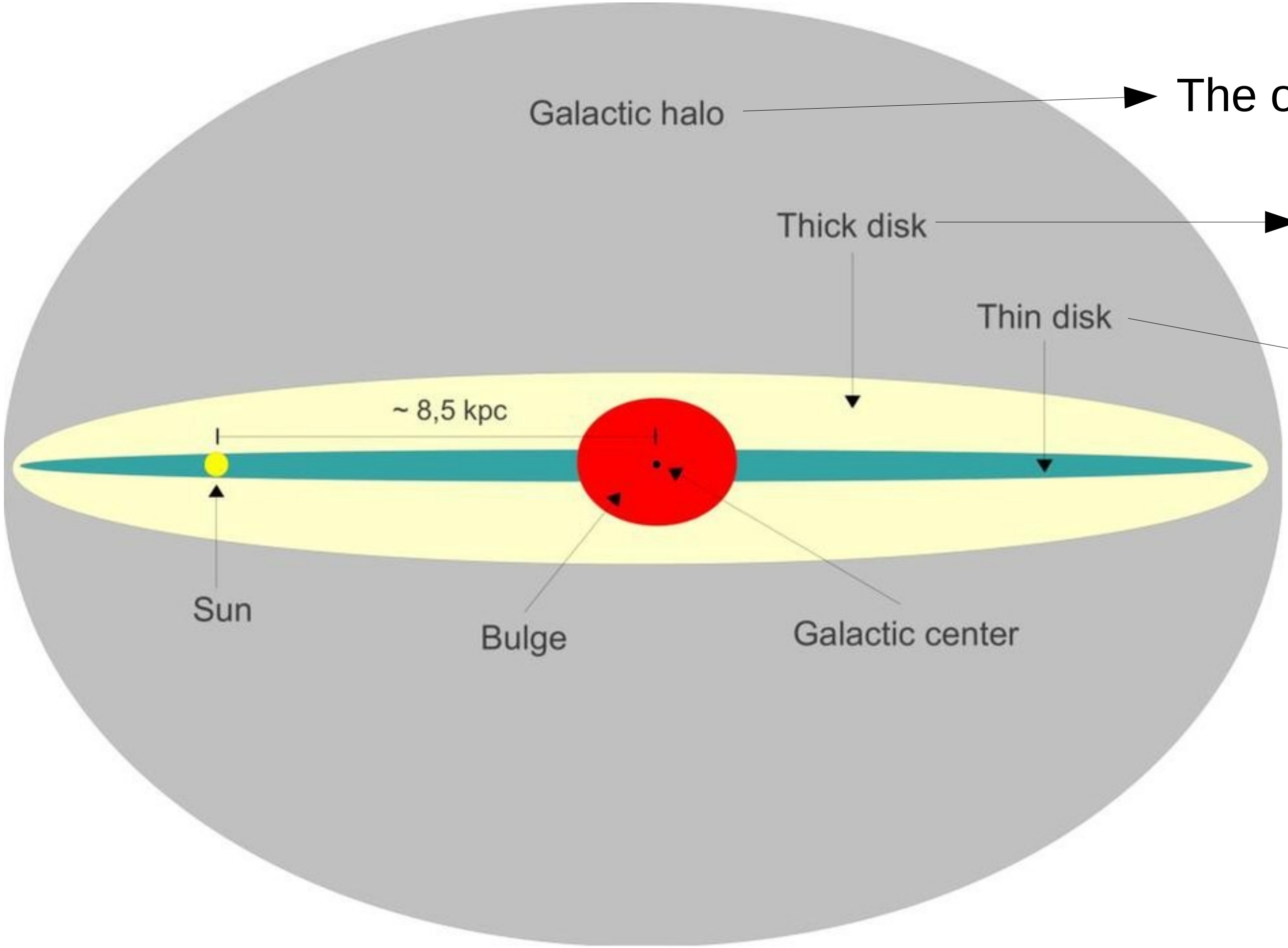
Constructing an H-R diagram



Constructing an H-R diagram



Anatomy of the Milky Way

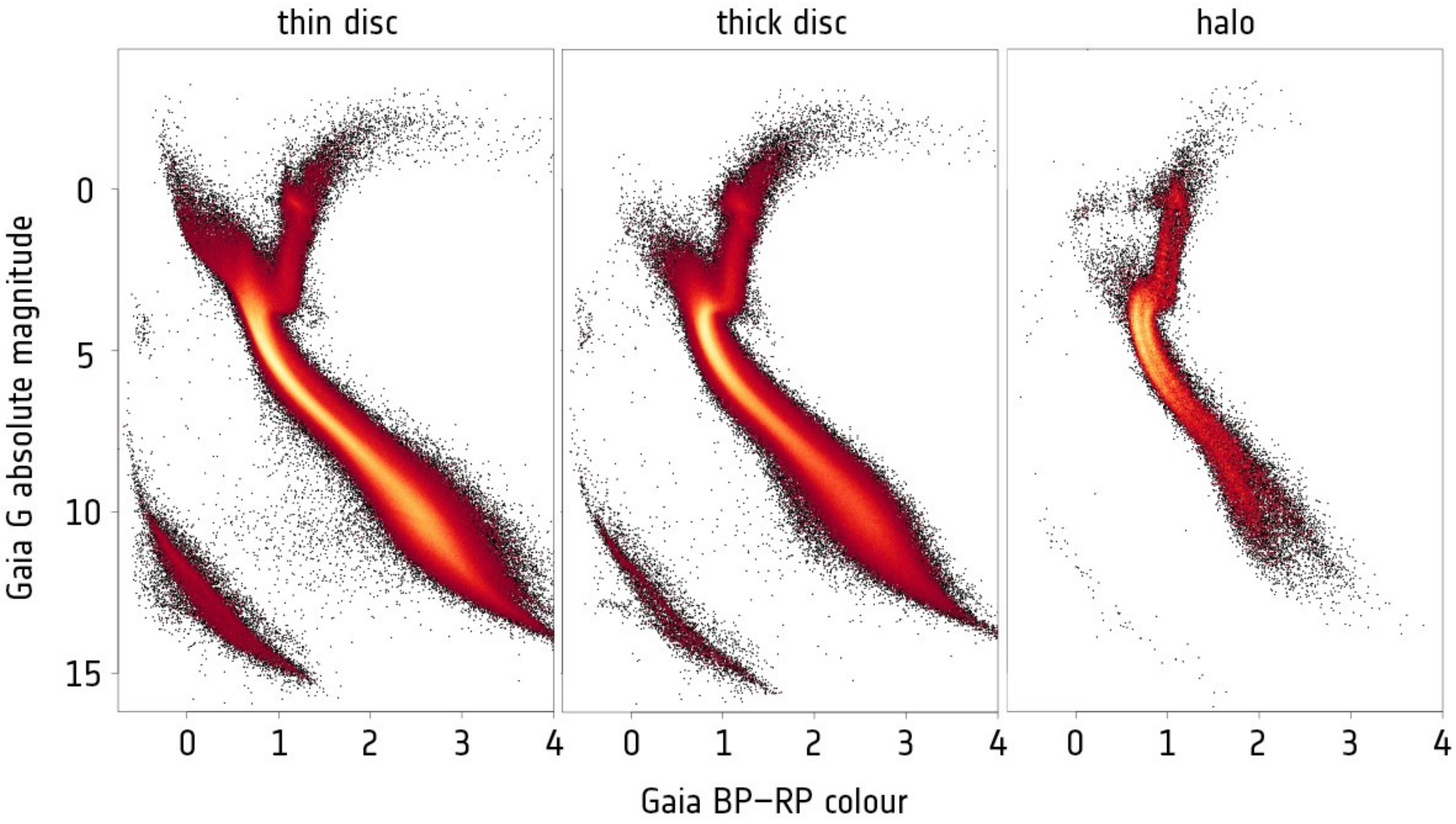


The oldest stars

Older stars, fewer metals

Younger, metal-rich stars

Gaia H-R diagrams of different parts of the Milky Way



Clusters

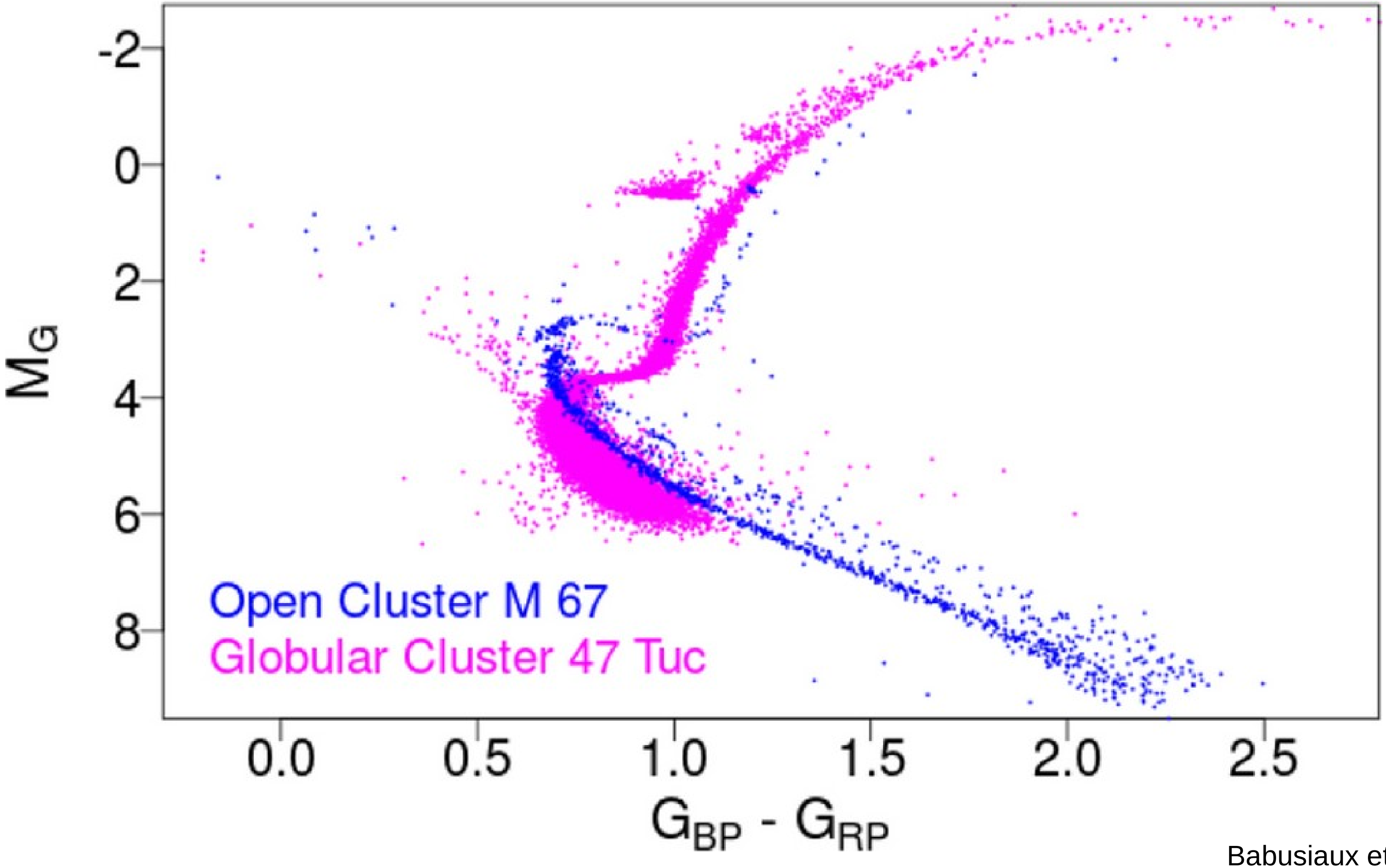
M13

M44

Globular Clusters (GCs): larger, denser and more massive, generally older, with less metals, found in the halo

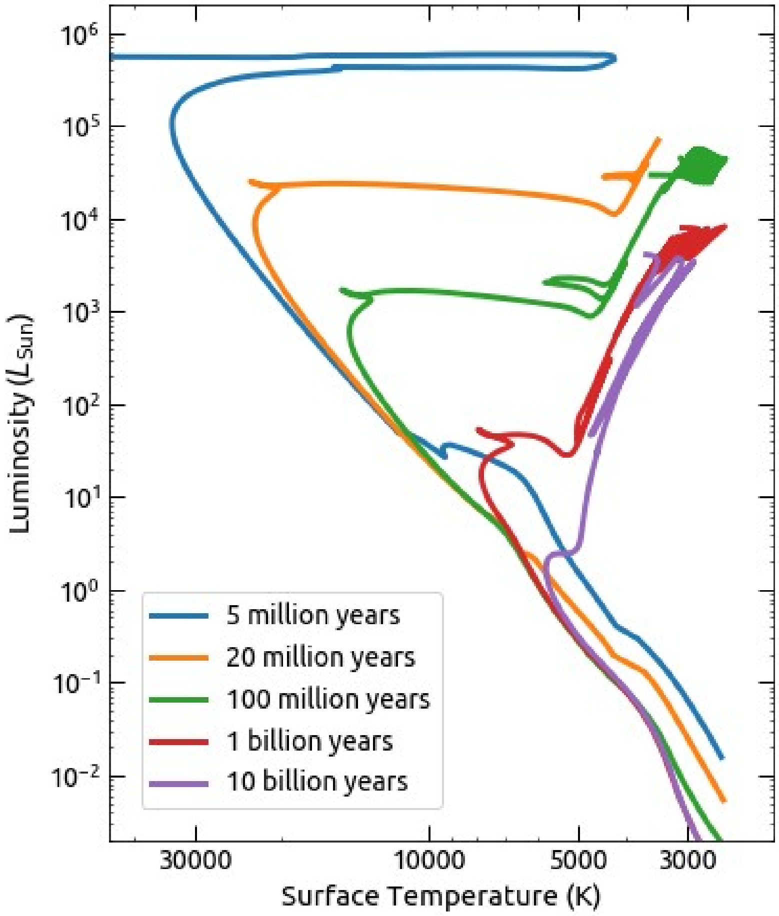
Open Clusters (OCs): less dense and less massive, generally younger and with more metals, found in the disk

H-R diagrams of Clusters



Babusiaux et al. 2018, A&A, 616

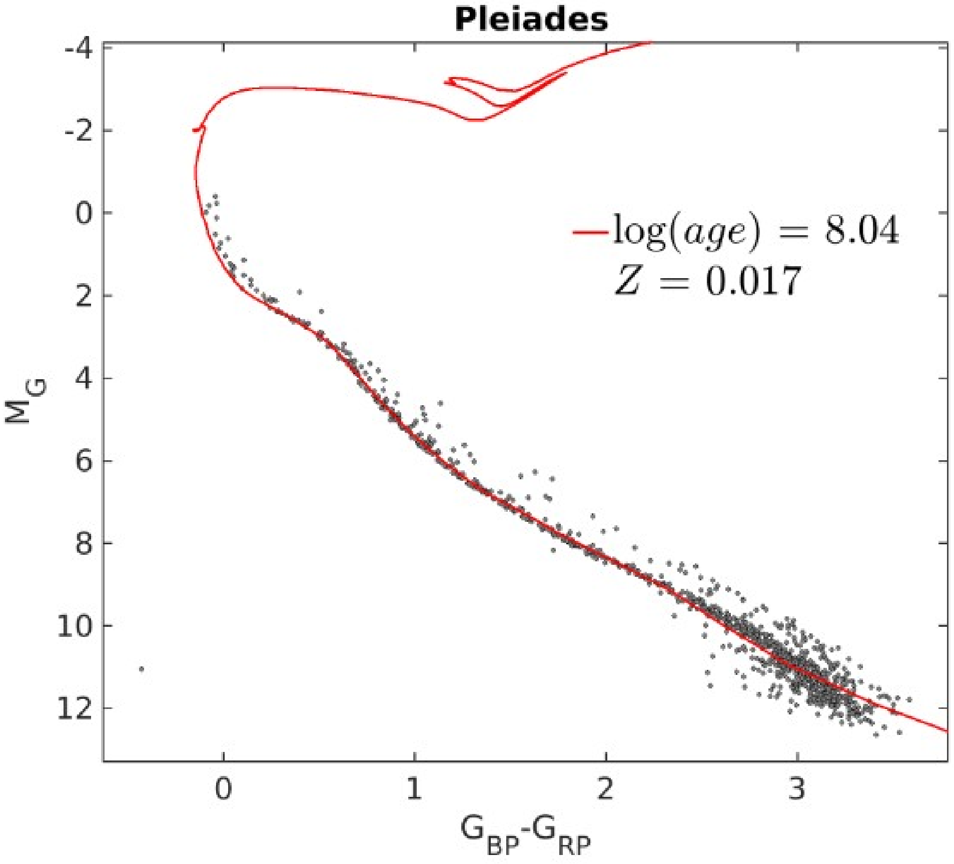
Modelling the H-R diagrams of clusters

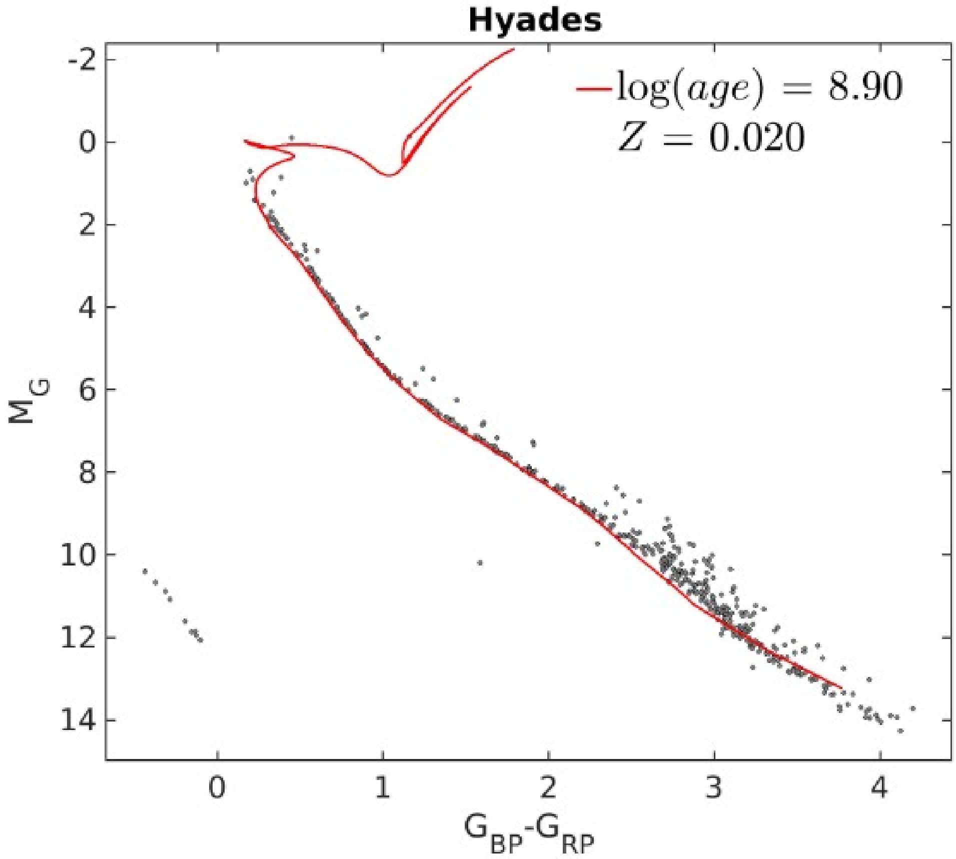


In the context of stellar evolution, a *stellar isochrone curve*, or simply ***isochrone***, is a curve on the H-R diagram representing a population of stars of *different mass* but of the *same age.*

Fitting an isochrone to the data of a cluster can provide the cluster's age, under the assumption that the stars of the cluster have been formed at the same time (which is generally true)

Modelling the H-R diagrams of clusters





SUMMARY

- We defined the luminosity and the brightness of a star, as well as its apparent and absolute magnitudes.

- We saw how the light emitted from a star follows Planck’s law. We also saw that stellar spectra have additional features, the spectral lines, associated with the presence of various elements, and that fitting models to the spectra can provide information on the physical properties.

- We saw how we can obtain a set of magnitudes using astronomical filters and how these are connected to temperature. We also saw that combining broadband and narrowband filters can reveal the presence of spectral lines and allow us to discover interesting objects.

- We defined and explored the Hertzsprung-Russell diagram.

- We saw how we can construct an H-R diagram and how we can obtain an estimate of the age of a cluster using stellar isochrones.